DIGITAL TECHNICS I

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4. LECTURE: SIMPLIFICATION AND MINIMIZATION OF LOGIC FUNCTIONS

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4. LECTURE

1. Canonical algebraic forms of logic functions (emphasis)
2. Minterm and maxterm tables
3. Simplification (minimization) of logic fuctions
4. Algebraic simplification of logic functions
5. Graphic representation (mapping) of logic functions and minimization

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CANONICAL FORMS OF LOGIC FUNCTIONS: EMPHASIS

It is expedient to base the synthesis of combinational circuit on the algebraic form of the logic function to be realized. Because a logic function can have several equivalent algebraic forms, the basis of the synthesis is one of the canonical forms (extended SOP or extended POS forms).

The disjunctive canonical form (extended sum-of-product, SOP) is given as a sum of conjunctive terms, i.e. minterms.

The conjunctive canonical form (extended product-of-sum, POS) is given as a product of disjunctive terms, i.e. maxterms.

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MINTERMS AND MAXTERMS

Writing the indices of minterms and of maxterms (function operators) in binary form, and associating a variable with each binary place in asserted or negated form, the algebraic form of the corresponding function operators will be obtained.

Example:

$m_5^5 = \overline{A}\,\overline{B}\,C\,\overline{D}\,E$ $(5 = 00101_{bin})$

$M_{12}^4 = A+B+\overline{C}+\overline{D}$ $(12 = 1100_{bin})$

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DISJUNCTIVE CANONICAL FORM EXTENDED SOP

Disjunctive canonic form (or extended sum-of-product form): Algebraic form consisting of sum of logic product terms (AND-OR) having the distinctive property that in each product term all variables are contained either in asserted or in negated form.

E. g.

$$F(ABC) = \overline{A}B\overline{C} + \overline{A}BC + \overline{A}\,\overline{B}C + \overline{A}BC$$

$$F(ABC) = m_2^3 + m_3^3 + m_4^3 + m_6^3$$

$$F = \Sigma^3(2,3,4,6)$$

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CONVERSION FROM SOP TO POS FORM

To obtain the conjunctive canonical form of the function $F = \Sigma^3(2,3,4,6)$ at first constitute the negated function

$$\overline{F(ABC)} = \Sigma^3(0,1,5,7) = \overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,C + \overline{A}\,B\,\overline{C} + ABC$$

$$\overline{F(ABC)} = m_0^3 + m_1^3 + m_5^3 + m_7^3$$

The negated function contains those minterms which are not contained in the function itself. Then apply a transformation based on De Morgan’s theorem!
(In the case of incompletely specified functions the situation is a bit more complicated.)

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CONJUNCTIVE CANONICAL FORM EXTENDED POS

Conjunctive canonic form (or extended product-of-sum form): Algebraic form consisting of product of logic sumt terms (OR-AND) having the distinctive property that in each sum term all variables are contained either in asserted or in negated form.

$$\overline{\overline{F(ABC)}} = F(ABC) =$$

$$(A + B + C)\,(A + B + \overline{C})\,(\overline{A} + B + \overline{C})\,(\overline{A} + \overline{B} + \overline{C})$$

$$F(ABC) = M_7^3 M_6^3 M_2^3 M_0^3$$

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CONNECTION BETWEEN MINTERMS AND MAXTERMS

Original function, disjunctive canonical form

$$F(ABC) = m_2^3 + m_3^3 + m_4^3 + m_6^3$$

Negated function, disjunctive canonical form (index i)

$$\overline{F(ABC)} = m_0^3 + m_1^3 + m_5^3 + m_7^3$$

Original function, conjunctive canonical form (index I)

$$F(ABC) = M_7^3 M_6^3 M_2^3 M_0^3$$

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DISJUNCTIVE CANONIC FORM

Values of the function: x_i (0 or 1).

n-variable function (minterms m_i^n, and $k = 2^n-1$)

$$F(A,B,C...) = x_0 m_0^n + x_1 m_1^n + ... + x_k m_k^n = \sum_{i=0}^{k} x_i m_i^n$$

The disjunctive canonic form (extended sum-of-products) contains those terms the values of which are 1.

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CONJUNCTIVE CANONIC FORM

Maxterms M_i^n

$$F(A,B,C...) = (x_0 + M_k^n)(x_1 + M_{k-1}^n) ... (x_k + M_0^n)$$

$$= \prod_{i=0}^{k} (x_i + M_{k-i}^n)$$

In the conjunctive canonical form (product-of-sums) those terms are present for which the value of the function is 0.

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CONNECTION BETWEEN MINTERMS AND MAXTERMS

All minterm is the inverse of a maxterm and vice versa.

$k = 2^n - 1$ and

$$\overline{m_i^n} = M_{k-i}^n$$

and

$$M_i^n = \overline{m_{k-i}^n}$$

The indices of minterms and maxterms, i és 2^n-1-i are the complements of each other.
In their binary forms the digits 0 and 1 are interchanged. The sum of the pairs of indices is 2^n-1, which in binary form contains only the digit 1.

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MINTERMES AND MAXTERMES

The sum of all the minterms of an n-variable function is 1, the product of all the maxterms is

$$\sum_{i=0}^{k} m_i^n = 1 \quad \text{and} \quad \prod_{i=0}^{k} M_{k-i}^n = 0$$

$(k = 2^n-1)$

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SPECIFICATION OF LOGIC FUNCTIONS

The disjunctive and conjunctive canonic forms can be designated with the operation symbols Σ, Π and acting on the indices of minterms and maxterms respectively. E.g.

$$f(A,B,C) = \bar{A}\,\bar{B}\,C + \bar{A}\,\bar{B}\,C + A\,B\,C = \Sigma^{3}(1, 4, 7)$$

$$f(A,B,C) = \Pi^{3}(1, 2, 4, 5, 7) =$$

$$(\bar{A} + \bar{B} + C)\,(\bar{A} + B + \bar{C})\,(A + \bar{B} + \bar{C})\,(A + \bar{B} + C)\,(A + B + C)$$

Remember: the disjunctive form doesn’t contain the indices 0,2,3,5, and 6, and their complements will be present in the conjunctive form.

INCOMPLETELY SPECIFIED LOGIC FUNCTION

A	B	C	F
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	-
1	0	1	0
1	1	0	-
1	1	1	1

The algebraic form of the incompletely specified logic function is written as

$$F(ABC) = \bar{A}\,\bar{B}\,\bar{C} + \bar{A}\,B\,\bar{C} + \bar{A}\,B\,C + A\,B\,C$$

$$+ (A\,\bar{B}\,\bar{C}) + (A\,B\,\bar{C})$$

Don’t care terms

$$F = \Sigma^{3}((0, 2, 3, 8) + (4, 6))$$

$$F = \Sigma^{3}((0, 2, 3, 8)\ X{:}(4, 6))$$

GRAPHIC MAPPING

The values of logic function can be represented in graphic forms (maps) based on the truth table. Mapping can be applied both to minterms and maxterms as well.

Important specific function maps are the *Karnaugh*-maps and the *Veitch*-diagrams.

In the case when the number of independent variables is limited, these representations are very expressive and useful, and can be very effectively applied in function minimization, etc.

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MINTERM AND MAXTERM TABLES

n = 4

A

0	2	10	8
1	3	11	9
5	7	15	13
4	6	14	12

D

B

C

Minterm table for n = 4.

Each cell corresponds to a minterm with the appropriate index. The lines beside the labels indicate 1, no line means 1.

E.g. for cell with decimal index 7 (m_7^4) , binary 0111

A = 0, B = 1, C =1, D =1.

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MINTERM AND MAXTERM TABLES

n = 4

A

15	13	5	7
14	12	4	6
10	8	0	2
11	9	1	3

D, B, D, C, C

Maxterm table for n = 4

The 3rd cell in the 1st row of the maxterm table is 5, while in the minterm table it is 10. These are the complements of each other, their sum is

$5 + 10 = 2^n - 1 = 2^3 - 1 = 15.$

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MINTERM AND MAXTERM TABLES

n = 4

A

0	2	10	8
1	3	11	9
5	7	15	13
4	6	14	12

D, B, C

A

15	13	5	7
14	12	4	6
10	8	0	2
11	9	1	3

D, B, D, C, C

The 3rd cell in the 1st row of the maxterm table is 5, while in the minterm table it 10. They are the complements of each other, their sum is 2^3-1 = 15.

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MINTERM TABLE N = 5

A

E E

	0	**1**	**5**	**4**	**16**	**17**	**21**	**20**
D	**2**	**3**	**7**	**6**	**18**	**19**	**23**	**22**
D	**10**	**11**	**15**	**14**	**26**	**27**	**31**	**30**
	8	**9**	**13**	**12**	**24**	**25**	**29**	**28**

B

C C

E. g.: $m_{19}^{5} \rightarrow 10011 \rightarrow A\,\bar{B}\,\bar{C}\,D\,E$

The difference of indices on the left and right half of the table is 16 corresponding to the weight of variable A.

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SIMPLIFICATION OF LOGIC FUNCTIONS: MINIMIZATION

Economy of a logic design:

1. Reduction of the number of gates;
2. Reduction of the number of interconnections;
3. Optimal choice of building blocks.

No general method exist for the solution of problem 3. For the solution of problems 1 and 2 there exist various solutions depending on the actual conditions.

In many cases the optimization (minimization) of a logic network can be posed as the minimization of the number of gate inputs. I.e. the cost function will be the number of gate inputs, or pins (pin count).

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LOGIC DESIGN: MINIMIZATION OF LOGIC FUNCTION

Aim: To find the most economic or cheapest implementation of the specified combinational network.

Specification: text, truth table, Boolean expression, canonical form, etc.

What is economical cheap? Depends on the „environment" (hardware base).

Discrete ICs (LSI/MSI, several gates in one IC): minimize the number of ICs, or the number of gates, or number of inputs (pins).

Programmable logic: minimization of the resources (logic cells) used.

VLSI: minimize the chip area, time delays, etc.

Simplest analysis: minimization of the number of inputs (pin count).

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SIMPLIFICATION GOALS

Goal -- minimize the cost of realizing a logic function.

Cost measures and other considerations:

- Number of gates
- Number of levels
- Gate fan in and/or fan out
- Interconnection complexity
- Preventing hazards

Two-level realizations:

- Minimize the number of gates (terms in logic function)
- Minimize the fan in (literals in logic function i.e. inputs/pins in gates)

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MINIMIMIZATION

In the past the main aim was to minimize the number of gate circuits implementing a given combinational circuit in order to decrease the number of electronic components.

Nowadays the main motivation for the minimization of logic network is to decrease the logic resources in a PLD of FPGA, to decrease the area in VLSIs, and to increase the operational speed and reliability of the circuits.

”The smallest number of failures are caused by those components which are NOT included in the network.” (Dr. Tóth Mihály, professor emeritus, Székesfehérvár.)

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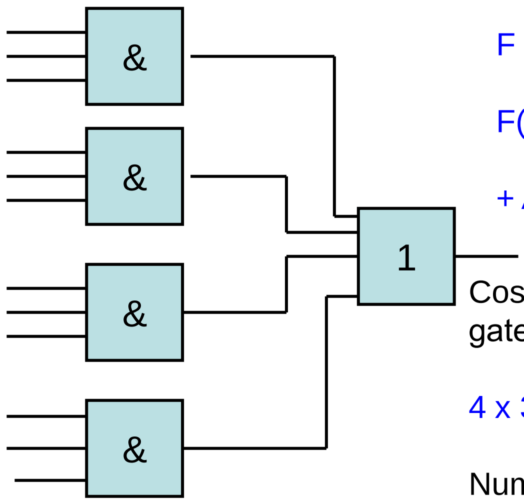
ILLUSTRATION: AND-OR REALIZATION OF F(A,B,C)

Implementation based on extended SOP form



$F = \sum^{3} (2,3,4,6) =$

$F(ABC) = \bar{A}B\bar{C} + \bar{A}BC$

$+ A\bar{B}\bar{C} + AB\bar{C}$

Cost function (number of gate inputs,”pin count”):

4 x 3 +1 x 4 = 16

Number of gates: 5

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ALGEBRAIC MINIMIZATION

$F(ABC) = \overline{A}B\overline{C} + \overline{A}BC + A\overline{B}\,\overline{C} + AB\overline{C}$

minterms $m_2^3 + m_3^3 + m_4^3 + m_6^3$

$= \overline{A}B(\overline{C} + C) + A\overline{C}(\overline{B} + B) = \overline{A}B + A\overline{C}$

The minterms which can be contracted differ only in one place:

(010) and (011), also (100) and (110)

This is the key of contraction and minimization!

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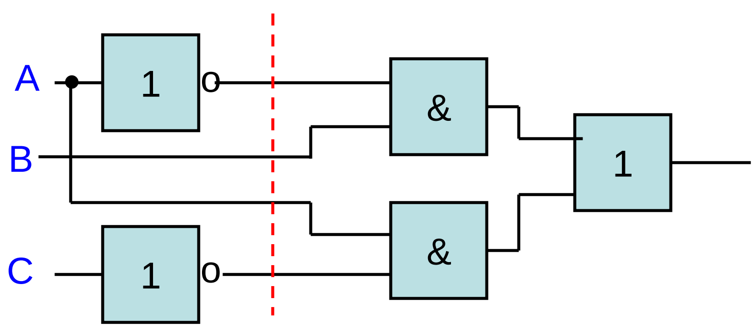
IMPLEMENTATION OF MINIMIZED FUNCTION



$F(A,B,C) = \overline{A}B + A\overline{C}$

Cost function (not including the inverters necessary to generate the negated input variables):
$2 \times 2 + 1 \times 2 = 6$ (before minimization:16 !)

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ADJACENT MINTERMS, MINIMIZATION

Adjacent minterms: only one logic variable asserted and negated respectively, all others are the same.

Process of contraction and minimization:
1. The adjacent minterms are contracted, the corresponding variables are eliminated.
2. In the new form the adjacent terms are again contracted, etc.
3. The process is continued till from the terms obtained no more variables can be eliminated by further contraction.

The terms obtained such way are called prime implicants of the function.

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MINIMAL FORM, PRIME IMPLICANTS

The minimized (disjunctive) form of the logic function is a sum of prime implicants.

$$F(ABC) = \overline{A}B\overline{C} + \overline{A}BC + A\overline{B}\,\overline{C} + AB\overline{C}$$

$$F(ABC) = \overline{A}B + \overline{A}C$$

here the prime implicants are $\overline{A}B$ and $\overline{A}C$

The aim of the minimization is to find the prime implicants of the function. It is to be remembered, that a logic function can have several equivalent simplest form!

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MINIMIZATION IN CONJUNCTIVE FORM

Similar approach can be applied to the conjunctive form (maxterms) of the functions. Two adjacent maxterms can be contracted to a single sum which will not contain the variable contained in negated and asserted forms in the two original maxterms. E.g.:

$$(\overline{A} + \overline{B} + \overline{C})(A + \overline{B} + \overline{C}) = (\overline{A} + (\overline{B} + \overline{C}))(A + (\overline{B} + \overline{C}))$$

$$= A\overline{A} + (A + \overline{A})(\overline{B} + \overline{C}) + (\overline{B} + \overline{C}) = \overline{B} + \overline{C}$$

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MINIMIZATION METHODS OF LOGIC FUNCTIONS

1. Minimization using Boolean algebraic transformations.
2. Algebraic minimization using Quine’s method.
3. Graphic minimization: using the Karnaugh-map.
4. Numeric (tabular) minimization: Quine-McCluskey-method.
5. ”Exact” algebraic methods, adaptable to computers, e.g. irredundant covering algorithm group.
6. Heuristic algorithms, (e.g. algorithms like PRESTO or ESPRESSO).

Here the algebraic (Quine’s) method, the graphic method (Karnaugh mapping) and the numeric (tabular) or Quine-McCluskey method will be described. The main attention will devoted to the graphic/mapping technique.

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QUINE'S METHOD

Quine's method helps to perform the algebraic minimization in a systematic and (hopefully) error-free way.

The method consists of factoring out the common factors from pairs of minterms in such a way that in the parentheses only the sum of one variable and its complement remained, which sum is trivially 1.
The process is applied to all possible pairs, then it is repeated with the new sets of terms, etc.

The method thus generates all prime implicants, therefore in the second phase the essential prime implicants should be selected to obtain the minimal cover.

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QUINE'S METHOD: DEMO

$F(A,B,C) = \overline{A}B\overline{C} + \overline{A}BC + A\overline{B}\,\overline{C} + AB\overline{C}$
(minterms: 2 3 4 6)

Minterms	I.	II.	III.
2	$\overline{A}B\overline{C}$ √	(2,3) $\overline{A}B$	No entries
3	$\overline{A}BC$ √	(2,6) $B\overline{C}$	
4	$A\overline{B}\,\overline{C}$ √	(4,6) $A\overline{C}$	
6	$AB\overline{C}$ √		

All prime implicants are contained in column II.

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QUINE’S METHOD: DEMO

Prime implicant table

	2	3	4	6
(2,3) $\overline{A}B$*	X	X		
(2,6) $B\overline{C}$	X			X
(4,6) $A\overline{C}$*			X	X

Identification of essential prime implicants to obtain the minimal cover

Minimal cover: $F(A,B,C) = \overline{A}B + A\overline{C}$

THE PEOPLE

Willard Van Orman Quine (1908-2000)

Spent his entire career teaching philosophy and mathematics at Harvard University, his alma mater, where he held the Edgar Pierce Chair of Philosophy from 1956 to 1978.

A computer program whose output is its source code is called a ”quine” named after him.

THE KARNAUGH MAP (K-MAP)

- It's nothing else, than a practical rearrangement of the truth table based on the concept of adjacency.
- The edges are labelled using a one-step (Gray) code.
- Each cell contains the actual value of the function belonging to the given combination of the independent variables.
- For the case of n variables the number of cells is 2^n.
- If the weights of the variables are agreed upon, individual indices can be rendered to each cell of the K map.

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THE KARNAUGH MAP (K-MAP)

The Karnaugh map, known also as Veitch diagram is a graphic tool to facilitate management of Boolean expressions.

It was invented in 1950 by *Maurice Karnaugh*, an engineer at Bell Labs, and independently by *Edward W. Veitch* in 1952.

Normally, extensive calculations are required to obtain the minimal form of a Boolean function.

Instead the Karnaugh maps make use of the human brain's excellent pattern matching capability to arrive at the simplest expression.

In addition, K-maps permit the rapid identification and elimination of potential race hazards, something that Boolean equations alone cannot do.

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THE PEOPLE

Maurice Karnaugh (1924-   ), American physicist.

Yale University, BSc. in maths and physics (1949), MSc. (1950), PhD in physics (1952) (*The Theory of Magnetic resonance and Lambda-Type Doubling in Nitric-Oxide*).

Bell Labs (1952-966), IBM (1966-1989).

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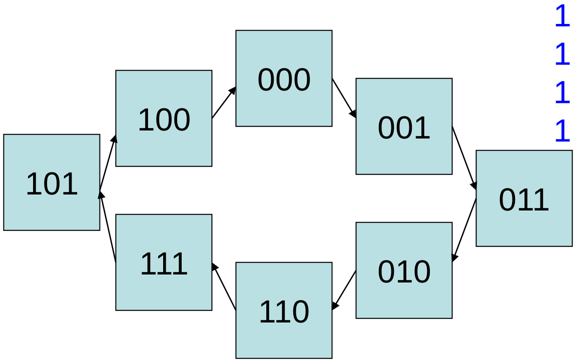
GRAY CODE

Gray code is a series of 2n codewords, each of n-bits, in such a sequence that any adjacent codewords differ only in one bit, including the first and last words too (cyclic property).
E. g. for n = 3 the sequence of codewords on the perimeter of the K-table:

A	B	C
0	0	0
0	0	1
0	1	1
0	1	0
1	1	0
1	1	1
1	0	1
1	0	0



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HAMMING DISTANCE

The Hamming distance of two codeword is the number giving that in how many places are the two codewords different.

The Hamming distance between any two adjacent codewords of the Gray code is one.

A	B	C
0	0	0
0	0	1
0	1	1
0	1	0
1	1	0
1	1	1
1	0	1
1	0	0

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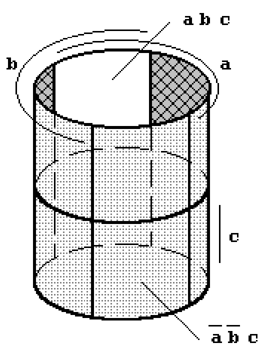
3-VARIABLE KARNAUGH MAP: ADJACENCY

A B	0	1	C
0 0	0	1	
0 1	2	3	
1 1	6	7	
1 0	4	5	

cell indices
(minterm indices)



The Hamming distances of the adjacent rows and columns are 1.

Rows and columns on the opposite sides are also adjacent (toroidal connections).

In the case of 5 or more variables this adjacency scheme becomes much more complex.

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EXAMPLE: FUNCTION MAPPING AND MINIMIZATION ON THE K-TABLE

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

$F = \Sigma\ (2,3,4,6)$

AB \ C	0	1
0 0	0	0
0 1	1	1
1 1	1	0
1 0	1	0

The adjacent cells (colour code) can be contracted (looped), and the simplest disjunctive form will be:

$$F = \overline{A}B + A\overline{C}$$

The terms obtained at the end of contracting process are the prime implicants.

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ADJACENCY AND CONTRACTION OF MINTERMS

The indices of adjacent cells differ only in the value of one variable, i.e. asserted or negated. The algorithm of the contraction is:

$$X \bullet (\text{anything}) + \overline{X} \bullet (\text{anything})$$

$$= (X + \overline{X}) \bullet (\text{anything}) = 1 \bullet (\text{anything}) = (\text{anything})$$

The adjacency relationships of the rows and columns of the Karnaugh table facilitates the easy identification of the possibilities of contractions.

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RULES OF CONTRACTION (LOOPING) ON THE KARNAUGH MAP

1. Adjacent cells can be contracted, in these terms contain a variable in asserted an in negated forms.

2. Cells on opposite edges can also be contracted (cyclicity).

3. Groups of areally connected cells comprising 2, 4, 8 ... 2^n, i.e. integer power of 2, cells can also be contracted.

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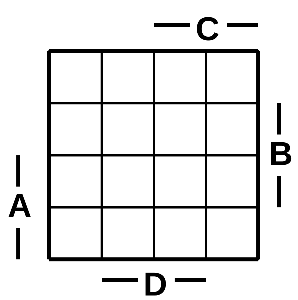
FOUR-VARIABLE KARNAUGH MAP

AB \ CD	0 0	0 1	1 1	1 0	
0 0	0	1	3	2	(0)
0 1	4	5	7	6	(4)
1 1	12	13	15	14	(12)
1 0	8	9	11	10	(8)
	(0)	(1)	(3)	(2)	

Akkor, és csak akkor ha
A→8; B→4; C→2; D→1;



Akkor, és csak akkor ha
A→8; B→4; C→2; D→1;

The labeling of the edges of the K map can be given by the binary combinations of the variables, or by lines indicating the asserted value of the appropriate variable.

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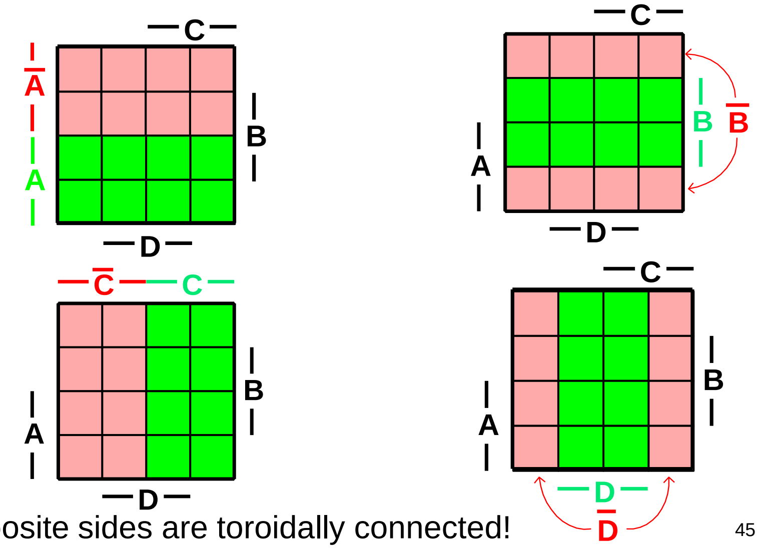
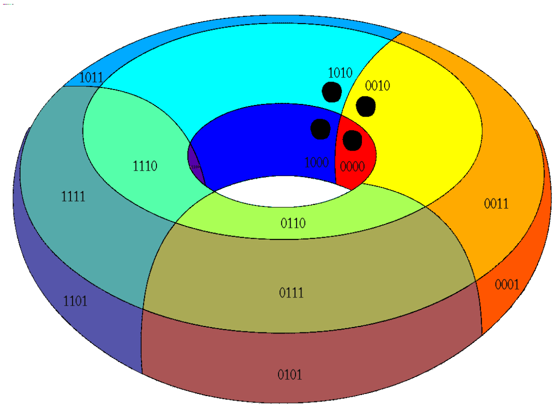
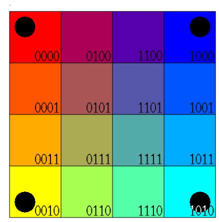
KARNAUGH MAP – GENERALIZATION OF THE VENN DIAGRAM



The opposite sides are toroidally connected!

FOUR VARIABLE K-MAP ON TOROID

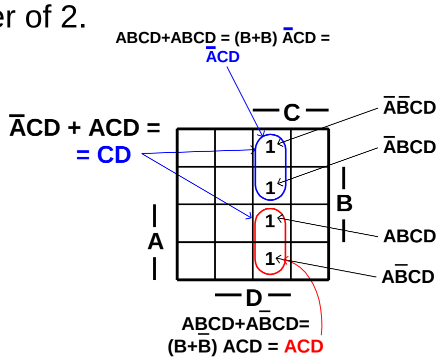




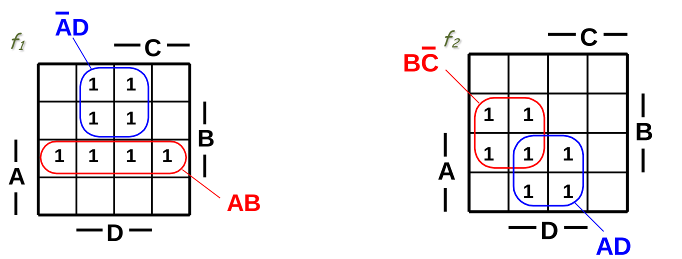
CONTRACTION EXAMPLES

Two adjacent cells or loops can always be contracted.

The number of cells in the resulting loop should always be an integer power of 2.



4-VARIABLE KARNAUGH MAP: COVERINGS (EXAMPLES)



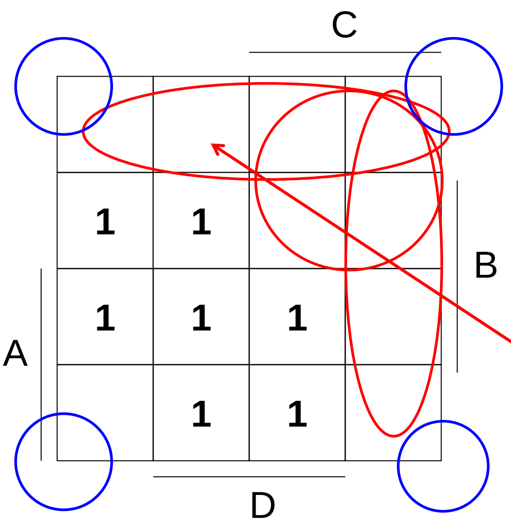
$$f_1 = \sum^{4}(1,3,5,7,12-15,) = AB + \overline{A}D$$

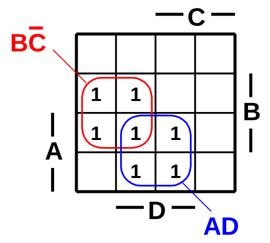
$$f_2 = \sum^{4}(4,5,9,11,12,13,15) = AD + B\overline{C}$$

Canonic forms minimized forms

Note: static hazard in f1 (map on the left)

MINIMIZATION IN CONJUNCTIVE FORM (PRODUCT-OF-SUMS, POS





$(A+B)(\overline{C}+D)(A+\overline{C})(B+D)$

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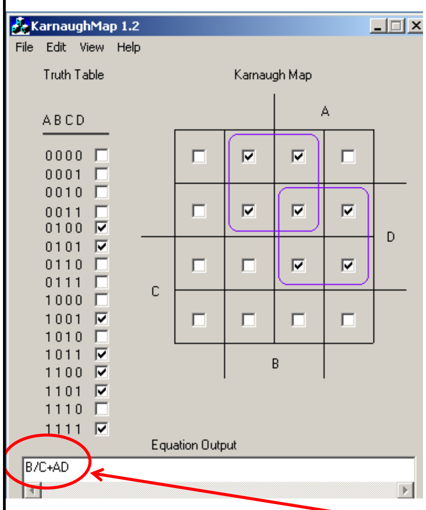
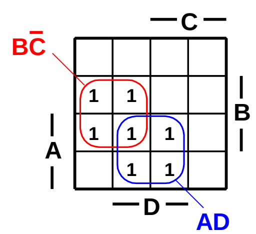
SUM-OF-PRODUCTS USING SOFTWARE





Implementation:

Two level AND-OR, or two-level NAND-NAND network

Minimal (simplest) form

$$f_2 = \sum^{4}(4,5,9,11,12,13,15) = AD + B\overline{C}$$

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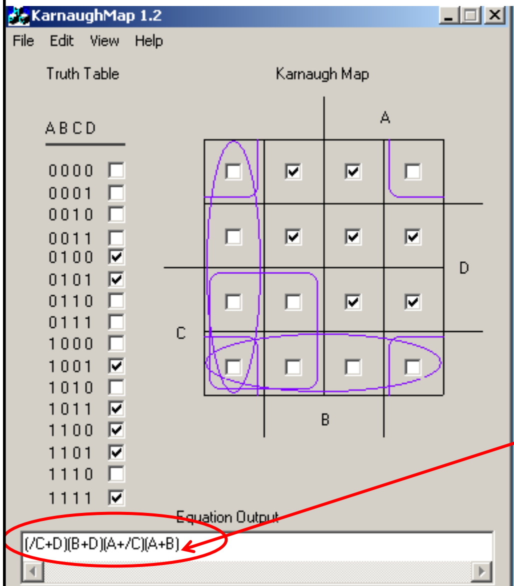
PRODUCT-OF-SUMS USING SOFTWARE



Implementation:

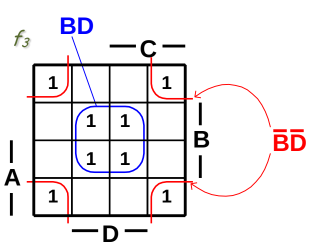
two-level OR-AND, or two-level NOR-NOR network

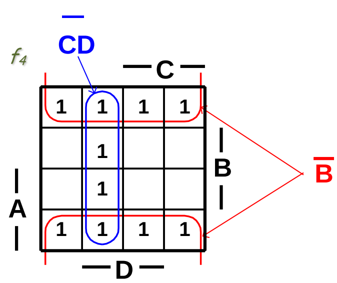
Minimal algebraic form

4-VARIABLE KARNAUGH MAP: COVERINGS (FURTHER EXAMPLES)

$$f_3 = \sum^4 (0,2,5,7,8,10,13,15) = BD + \overline{B}\,\overline{D}$$

$$f_4 = \sum^4 (0-3,5,8-11,13) = \overline{B} + C\overline{D}$$





Note: symmetric function f3, ”chess board” pattern (map on the left)

4-VARIABLE KARNAUGH MAP: AN OTHER ARRANGEMENET OF AXES

A

8	**12**	**4**	**0**
10	**14**	**6**	**2**
11	**15**	**7**	**3**
9	**13**	**5**	**1**

C

D

B

Different labelling of axes, the positions of minterms are changed, however the adjacency reamians unaltered.

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ADJACENCY UP TO FOUR VARIABLES

2 variables

a	a
a	

A

B

3 variables

a	a	b	a
a	b	**b**	b

A

C

B

4 variables

a	a	b	a
a			
		b	
a	b	**b**	b

B

A

D

C

	c		
c	**c**	c	
	c		

B

A

D

C

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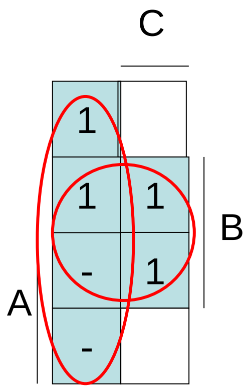
INCOMPLETELY DETEMINED FUNCTION ON THE KARNAUGH MAP

A	B	C	F
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	-
1	0	1	0
1	1	0	-
1	1	1	1

$F = \Sigma^3((0,2,3,7) + (4,6))$



Optimal cover two 4-loops

$F = B + \overline{C}$

Don't care terms are taken as 1s.

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INCOMPLETELY SPECIFIED LOGIC FUNCTION

During contraction the not specfied function values can be freely chosen as 1 or 0, depending on which choice would result in the most advantegeous solution.

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TWO-LEVEL IMPLEMENTATION OF COMBINATIONAL NETWORKS

The disjunctive canonic form (extended sum-of-product, SOP) results in a two-level realization (AND-OR structure).

The minimization results also in a two-level, however simpler network (less gates and less interconnections (i.e. pins)).
If the operation speed is critical, if possible, it is not advisable to increase the number of levels.

The propagation delays are added in the case of series connected gates!

A further important concern is the dynamic behaviour, especially the hazard phenomena.

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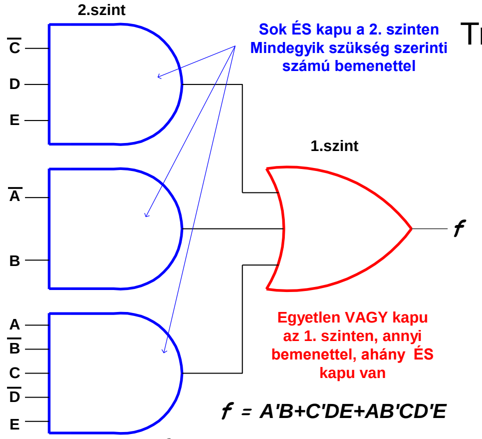
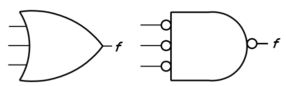
TWO-LEVEL AND-OR NETWORK



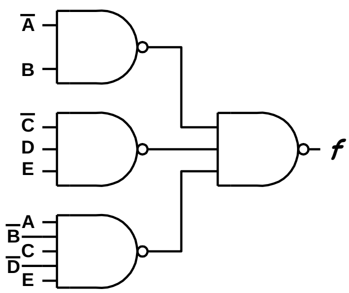
f = A'B+C'DE+AB'CD'E

f = Sum^5 (3, 8-15, 19, 21, 27)
A, B, C, D, E → 16, 8, 4, 2, 1

Transformation of output gate



$$\overline{A+B+C} = \overline{A}.\overline{B}.\overline{C}$$



Homogeneous NAND network 58

MAPPING AND MINIMIZATION WITH 5 OR MORE VARIABLES

In case of 5 variables the minimization can be perfomed using two 4-variable maps, for 6 variables using four 4-variable maps. A preferred alternative is to use the appropriate 5- or 6-varible Karnaugh maps.

The simultaneous handling of four tables is already difficult. For 6 or more variables minimization based on Karnaugh mapping is not practical.

For 5- and 6-variable Karnaugh maps (labelling) see: Rőmer p. 27, Zsom I p. 129, also in Arató.

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5-VARIABLE MINIMIZATION ON THE KARNAUGH MAP

Illustration of the method using two 4-variable maps

F(ABCDE) =

$$\Sigma^{5} (0,4,5,10,11,14,16,20,21,24,25,26,27,30)$$

It can be already seen that e.g. the minterms 24(=16+8, i.e. 11000),25 (i.e.11001), then 26,27 van be contracted , then the two resulting terms can also be combined, therefore both D and E drops out, etc.

(This example can be found in Arató p. 59., but note the printing errors in the algebraic form.)

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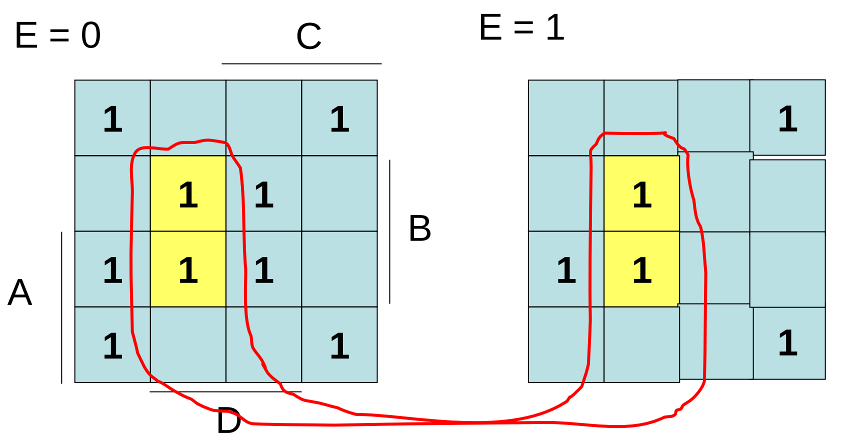
5-VARIABLE MINIMIZATION(1)



Prime implicant: $B \overline{C} D$

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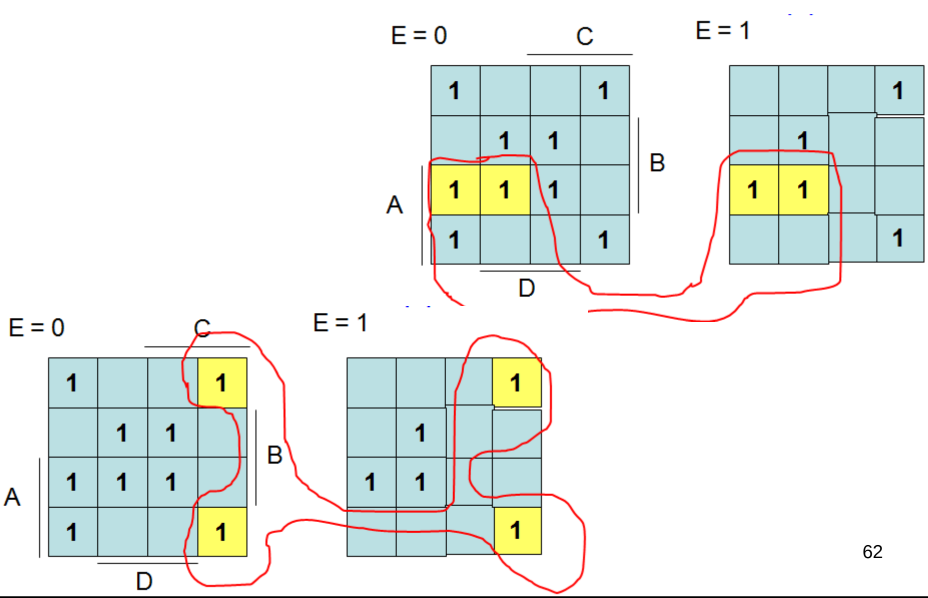
5-VARIABLE MINIMIZATION (2,3)



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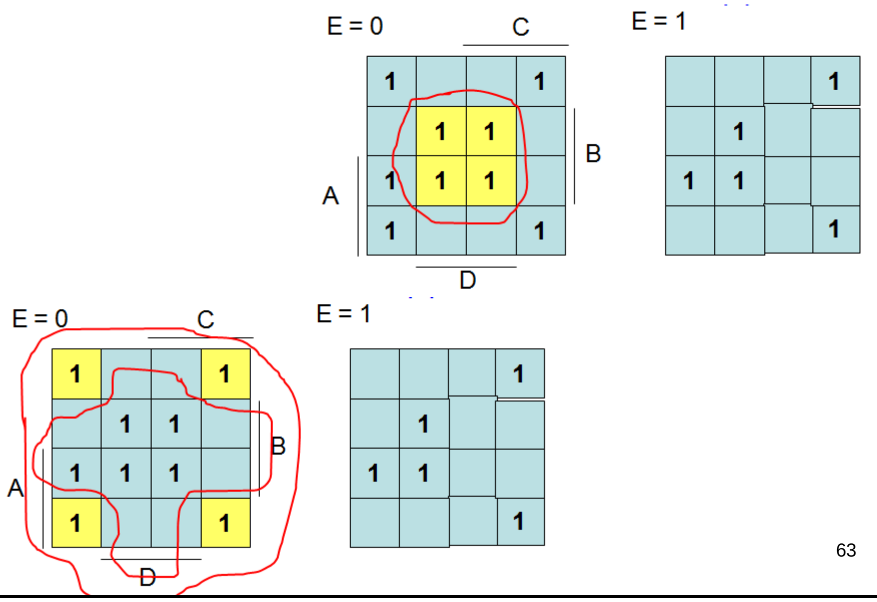
5-VARIABLE MINIMIZATION (4,5)



SIMPLIFIED (OPTIMIZED) FORM

Five prime implicants are in the minimized function

$F(ABCDE) = AB\overline{C} + B\overline{C}D + \overline{B}C\overline{D} + BD\overline{E} + \overline{B}\,\overline{D}\,\overline{E}$

In the original form, the total number of inputs is 14 x 5 + 14 = 84, while in the minimized form 5 x 3 + 5 = 20.

Gates: five 3-input NAND, one 5-input OR, and five inverters.

Packages (TTL series 74): one HEX INV, two 3-input NAND, one 8-input NAND.

5-VARIABLE KARNAUGH MAP

	C 0	0	0	0	1	1	1	1
	D 0	0	1	1	1	1	0	0
AB	E 0	1	1	0	0	1	1	0
0 0	0	1	3	2	6	7	5	4
0 1	8	9	11	10	14	15	13	12
1 1	24	25	27	26	30	31	29	28
1 0	16	17	19	18	22	23	21	20

The two 4-variable K-maps can be merged by appropriate changes in the labeling. This can be done in several different ways. Here the so called reflection map is shown. In establishing adjacency, the reflection on the vertical symmetry axis should also be considered.

IN PLANE KARNAUGH MAP FOR 5 VARIABLES

Adjacency:

9 →0 1 0 0 1
13→0 1 1 0 1

Reflection symmetry

	C 0	0	0	0	1	1	1	1
	D 0	0	1	1	1	1	0	0
AB	E 0	1	1	0	0	1	1	0
0 0	0	1	3	2	6	7	5	4
0 1	8	$\boxed{9}$	11	10	14	15	$\boxed{13}$	12
1 1	24	25	27	26	30	31	29	28
1 0	16	17	19	18	22	23	21	20

For a 3-dimenional arrangement of the 5-variable K-map and looping on it cf. Zsom Vol. 1, p.129

5-VARIABLE IN-PLANE KARNAUGH MAP

D

C

0	**1**	**3**	**2**	**6**	**7**	**5**	**4**
8	**9**	**11**	**10**	**14**	**15**	**13**	**12**
24	**25**	**27**	**26**	**30**	**31**	**29**	**28**
16	**17**	**19**	**18**	**22**	**23**	**21**	**20**

B

A

E E

In establishing adjacency the reflection symmetry is also accounted for. E.g. m9 (01001) and m13 (01101) are adjacent.

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5-VARIABLE IN-PLANE KARNAUGH MAP

D

C

0	**1**	**3**	**2**	**6**	**7**	**5**	**4**
8	**9**	**11**	**10**	**14**	**15**	**13**	**12**
24	**25**	**27**	**26**	**30**	**31**	**29**	**28**
16	**17**	**19**	**18**	**22**	**23**	**21**	**20**

B

A

E E

m10 (01010) adjacent cells: (the differences are 1, 2, 4, 8, and 16, i.e powers of 2.

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5-VARIABLE IN-PLANE KARNAUGH MAP

0	1	3	2	6	7	5	4
8	9	11	10	14	15	13	12
24	25	27	26	30	31	29	28
16	17	19	18	22	23	21	20

D, C, B, A, E, E

m9 is also adjacent to m13 (reflection symmetry!)

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5-VARIABLE IN-PLANE KARNAUGH MAP

0	1	3	2	6	7	5	4
8	9	11	10	14	15	13	12
24	25	27	26	30	31	29	28
16	17	19	18	22	23	21	20

D, C, B, A, E, E

The (left) half-map's edges are touching (toroidal connection!) therefore m2 is also adjacent to m0.

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5-VARIABLE IN-PLANE KARNAUGH MAP

D

C

0	**1**	**3**	**2**	**6**	**7**	**5**	**4**
8	**9**	**11**	**10**	**14**	**15**	**13**	**12**
24	**25**	**27**	**26**	**30**	**31**	**29**	**28**
16	**17**	**19**	**18**	**22**	**23**	**21**	**20**

B

A

E E

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ADJACENCY ON 5-VARIABLE MAP

a	a	d	a		c		a
a		c		c	**c**	c	b
b		d		b	c	b	**b**
a	d	**d**	d		d		b

B

A

D C

E E

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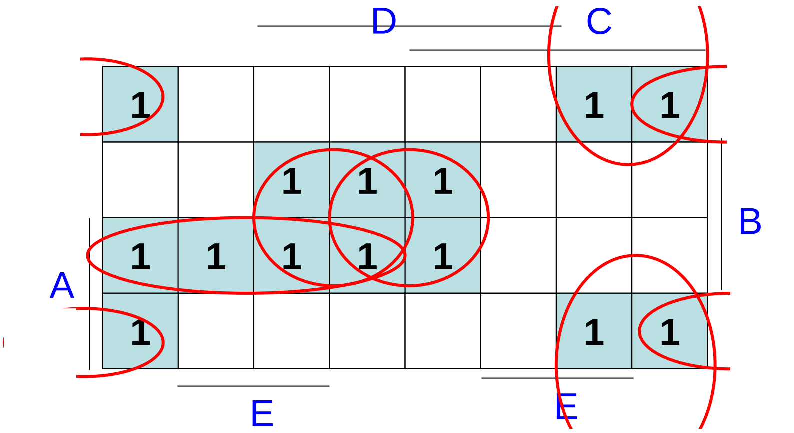
EXAMPLE: MINIMIZATION ON 5-VARIABLE K MAP

$F(ABCDE) = \Sigma^5(0,4,5,10,11,14,16,20,21,24,25,26,27,30)$



5 four-cube loops can be identified

$F(ABCDE) = AB\overline{C} + B\overline{C}D + \overline{B}C\overline{D} + BD\overline{E} + \overline{B}\,\overline{D}\,\overline{E}$

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END OF LECTURE

APPENDIX: 6-VARIABLE KARNAUGH TABLE AND MINIMIZATION

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6-VARIABLE KARNAUGH MAP

In the case of 6 variables four 4-variable Karnaugh map are necessary to represent the function. From the 6 variables the values of two selected ones should be fixed on each 4-variable map.

An alternative is to use an in-plane map with appropriately coded labeling on the its edges. E.g. a 8 by 8 construction is shown here.

ABC \ DEF	000	001	011	010	110	111	101	100
000	0	1	3	2	6	7	5	4
001	8	9	11	10	14	15	13	12
011	24	25	27	26	30	31	29	28
010	16	17	19	18	22	23	21	20
110	48	49	51	50	54	55	53	52
111	56	57	59	58	62	63	61	60
101	40	41	43	42	46	47	45	44
100	32	33	35	34	38	39	37	36

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KARNAUGH MAP FOR 6 VARIABLES

Adjacency:

0 0 1 0 0 1
0 0 1 1 0 1
1 0 1 0 0 1
1 0 1 1 0 1

ABC \ DEF	000	001	011	010	110	111	101	100
000	0	1	3	2	6	7	5	4
001	8	$\boxed{9}$	11	10	14	15	$\boxed{13}$	12
011	24	25	27	26	30	31	29	28
010	16	17	19	18	22	23	21	20
110	48	49	51	50	54	55	53	52
111	56	57	59	58	62	63	61	60
101	40	$\boxed{41}$	43	42	46	47	$\boxed{45}$	44
100	32	33	35	34	38	39	37	36

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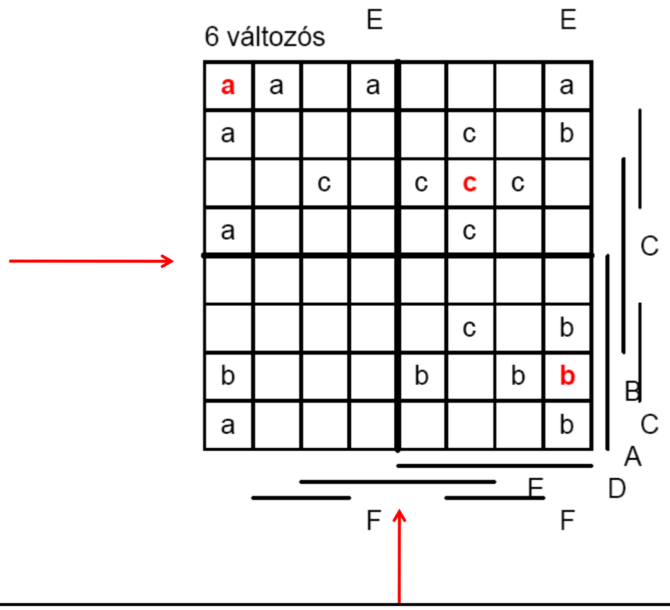
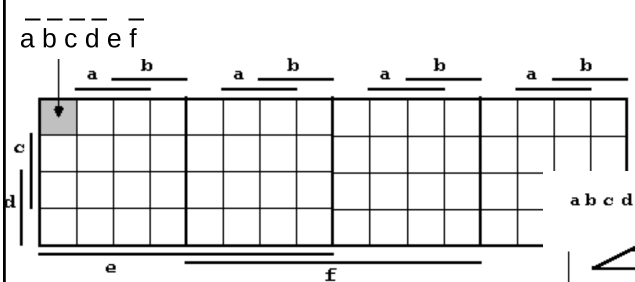
ADJACENCY FOR 6-VARIABLES

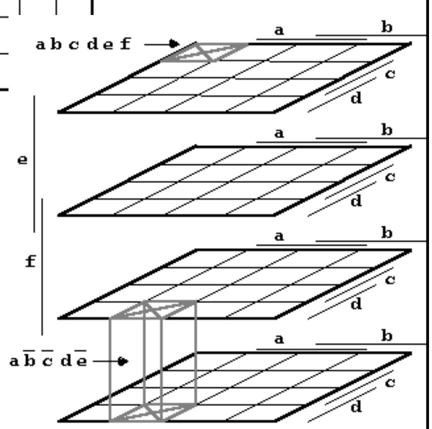


KARNAUGH MAP FOR 6-VARIABLES



Two dimensional (in-plane) arrangement of 6-variable Karnaugh map

Three dimensional arrangement of 6 variable K map



EXAMPLE: MINIMIZATION ON THE SIX-VARIABLE KARNAUGH MAP

Function to be minimized (19 minterms):

F(A,B,C,D,E,F) =

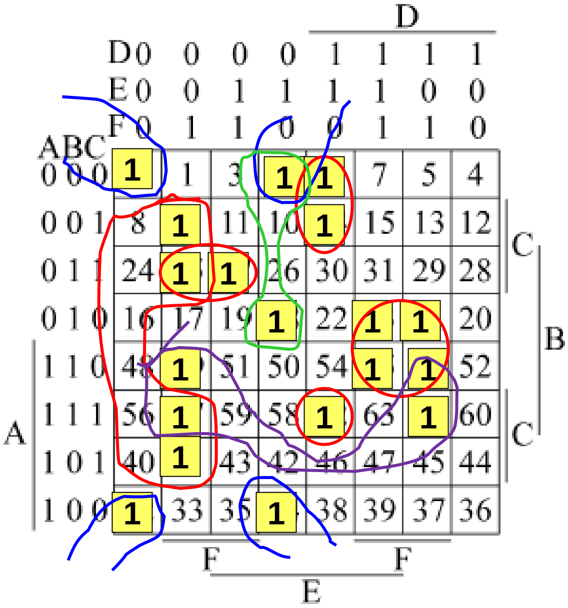
Σ^6 (0,2,6,9,14,18,21,23,25,27,32,34,41,49,53,55,57,61,62)

EXAMPLE: MINIMIZATION WITH SIX VARIABLES

ABC \ DEF	000	001	011	010	110	111	101	100
000	1	1	3	1	1	7	5	4
001	8	1	11	10	14	1	13	12
011	24	1	1	26	30	31	29	28
010	16	17	19	1	22	1	1	20
110	48	1	51	50	54	1	1	52
111	56	1	59	58	1	63	1	60
101	40	1	43	42	46	47	45	44
100	1	33	35	1	38	39	37	36

F(A,B,C,D,E) = Σ^6 (0,2,6,9,14,18,21,23,25,27,32,34,41,49,53,55,57,61,62)

EXAMPLE: LOOPING FOR SIX VARIABLES



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