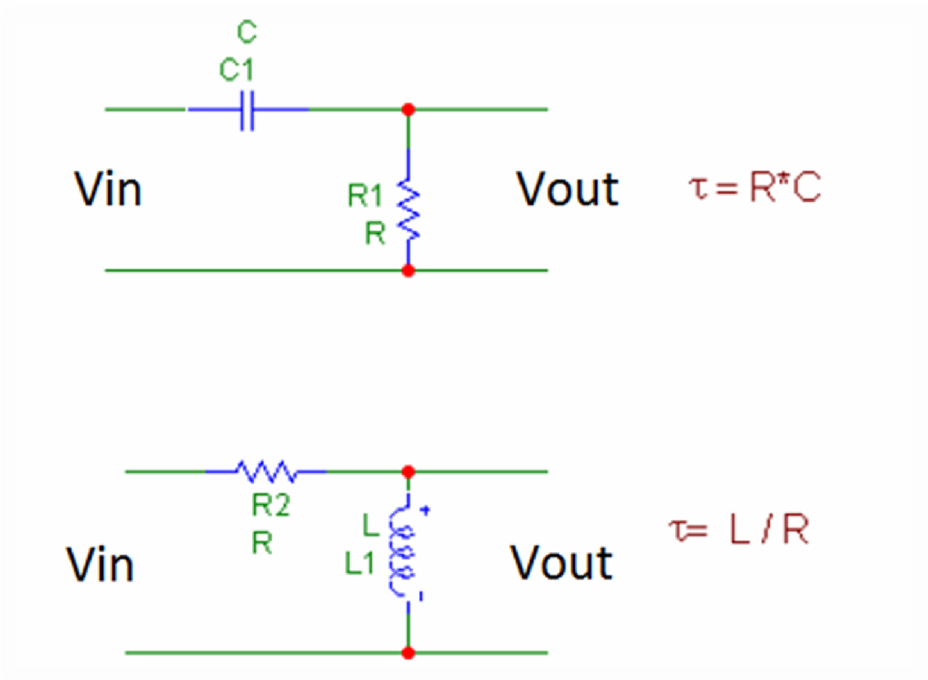
RLC four-pole investigation

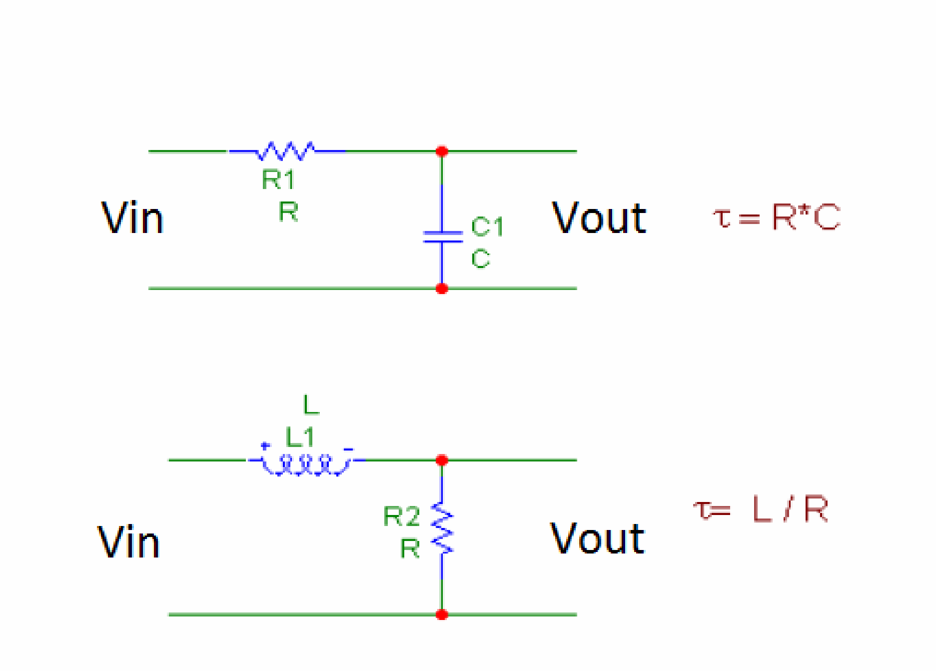
The most basic circuits contain resistors, capacitors and inductors. Circuits made of these elements are called passive circuits. RLC elements have an important role in modelling active elements with replacement method, mainly describing high frequency behaviour. RLC elements can be used as frequency dependent voltage dividers, integrating or differentiating circuits, but mostly as circuit unit components. Connected in series with operation amplifier's input, or placed in the feedback branch, we can get active filter circuits. We can use passiv elements to set broadband amplifier's transmission until 100MHz.

Elementary RC and RL four-poles:



Differentiating units

Figure No 1



Integrating units

Figure No 2

Figure to interpret the time constant:

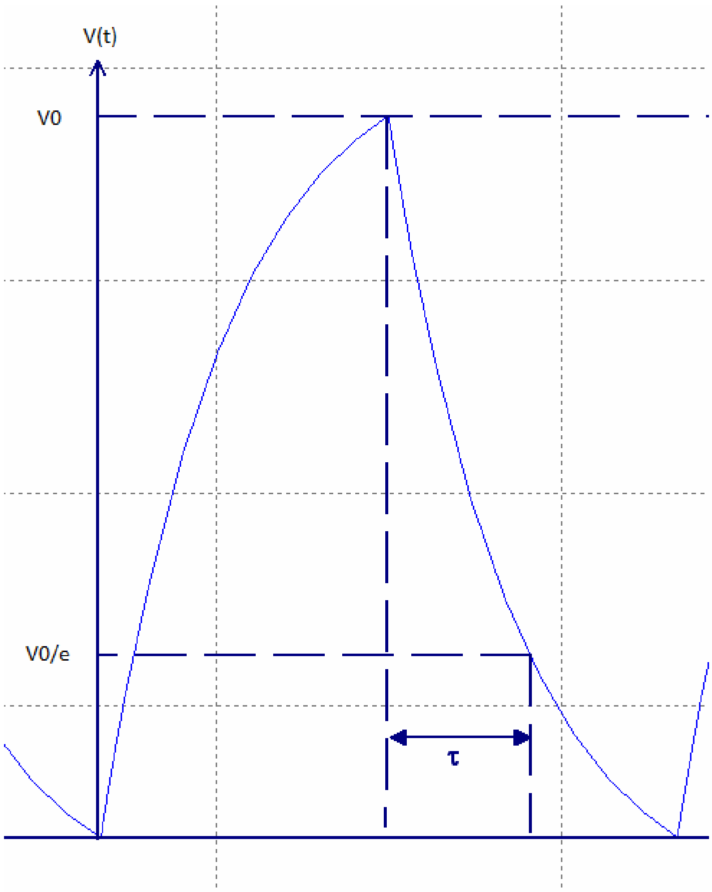


Figure No 3

As all electric circuit, we can examine the above examples in time interval and in frequency interval.

If we write the Kirchoff-equations in case of arbitrary propagation time currents and voltages, we get an integrating-defferentiating system of equation.

In case we examine in the aspect of time interval we induce the network with jump fuction.

In this case the solution of the differentiating equation describing the output voltage is an exponential time function.

Let us examine the **integrating** units' response signal while we connect steep up-and downward propagation voltage impulse to the input.

In case of integrating unit the response signal of the upward propagation:

$$V_{out}=V_0(1-e^{t/\tau}) \qquad (1)$$

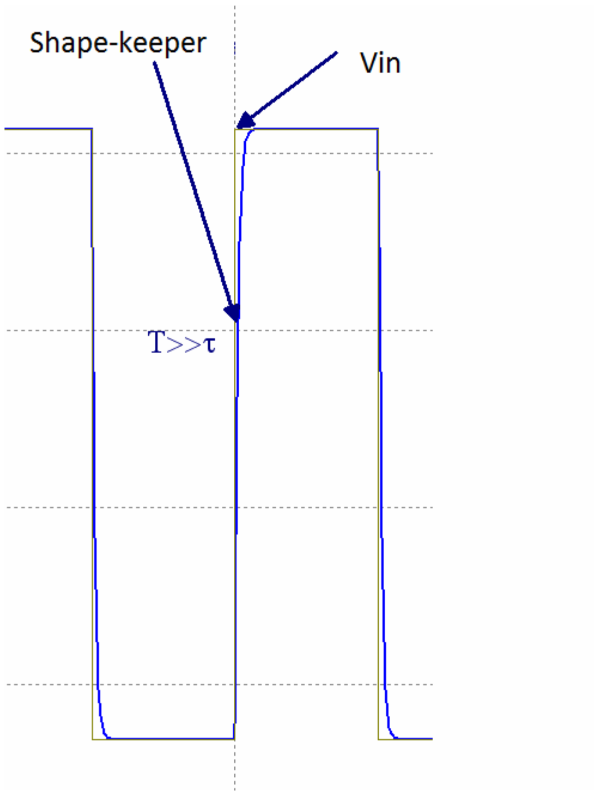
Downward propagation:

$$V_{out}=V_0*e^{t/\tau} \qquad (2)$$

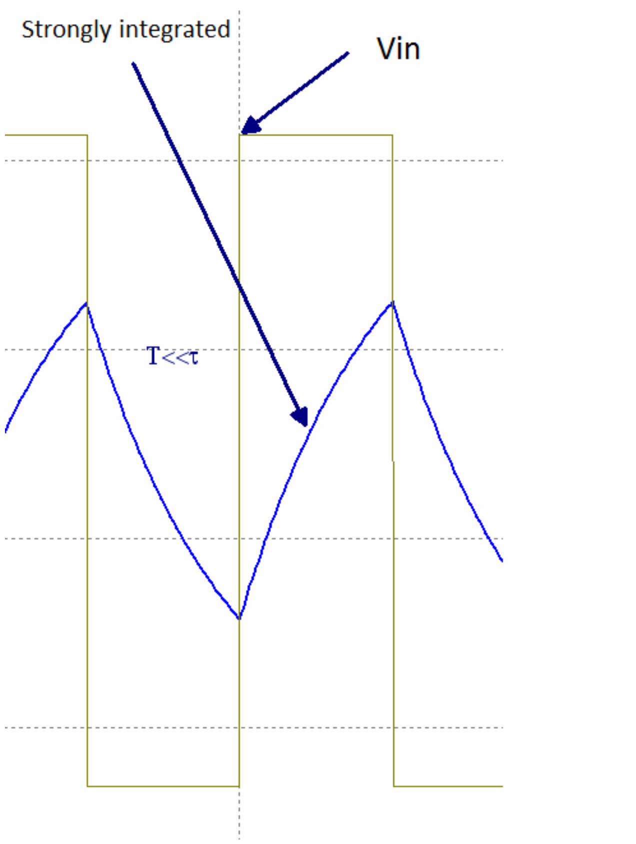
While interpreting the functions, switching on and off the input of V_0 jump function always occur at t=0 momentum.

As V_0 constant voltage (the amplitude of the impulse) integrated in the aspect of time results V_0*t, the circuit behaves as it would mathematically integrate the input voltage. This is why the circuits shown in figure No 2 are called integrating units.

In the case of **integrating unit**, if the inducing impulse series' **T period time is much bigger** than τ time constant (T>>τ), then the response signal V_{out} can be seen as shape-keeper.



In the case of **integrating unit**, if the inducing impulse series' **T period time is much smaller** than τ time constant (T<<τ), then the response signal V_{out} is distorted to triangle shape.



While examining in the aspect of frequency interval, we induce the circuit with sinusoidal propagation time signals. The most picturesque method when we describe in the aspect of complex frequency interval, which represents both the amplitude and phase relations. Examining the integrating units, at low frequency the circuit barely sates, while increasing the frequency the sating is increasing, and the amplitude of the output voltage decreases sharply (-20dB/dec).

Not detailing the amplitude and phase relations, the integrating unit behaves as a low-pass filter.

The behaviour in frequency interval can be easily connected with the behaviour in time interval. If $\tau \ll T$ (low frequency inducing), the transmission is shape-keeper and the and the sate is small. In case of $\tau \gg T$ the sate will be bigger. For this interval, the following expression is true:

$$V_{out} \;\square\; \text{constant} \int V_0 \sin\omega t \, dt \quad (3)$$

After performing the integration we get:

$$V_{out} \;\square\; \text{constant} \, (-V_0/\omega) \cos\omega t \quad (4)$$

So, the amplitude of the signal is decreasing reversely proportianally to ω. The constant in the expressions is proportional to R*C and R/L.

The behaviour in the aspect of frequency interval can be well examined with Bode-diagrams.

The integrating unit can be represented with a one-break point Bode- diagram, where the break point (the frequency belonging to the 3dB decreasing) is at $f_0 = 1/(2\pi*\tau)$ frequency and from this point the amplitude decreases with 20dB/dec steepness.

Differentiating units

We can interpret the behaviour of differentiating units in figure No 1 similarly.

Here, the response signal of the upward propagation:

$$V_{out} = V_0 * e^{t/\tau} \qquad (5)$$

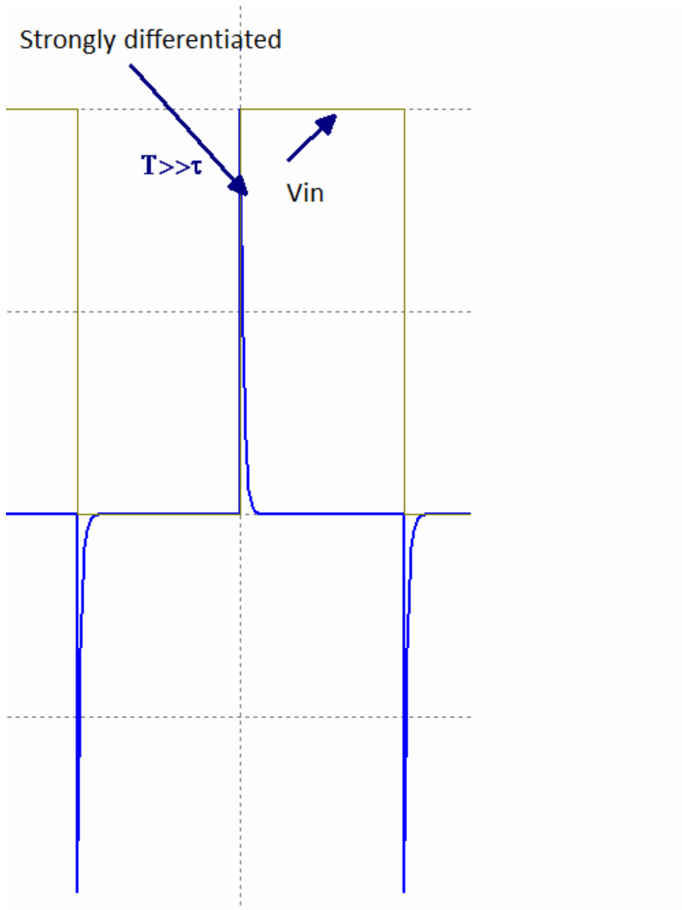
Downward propagation:

$$V_{out} = -V_0 * e^{t/\tau} \qquad (6)$$

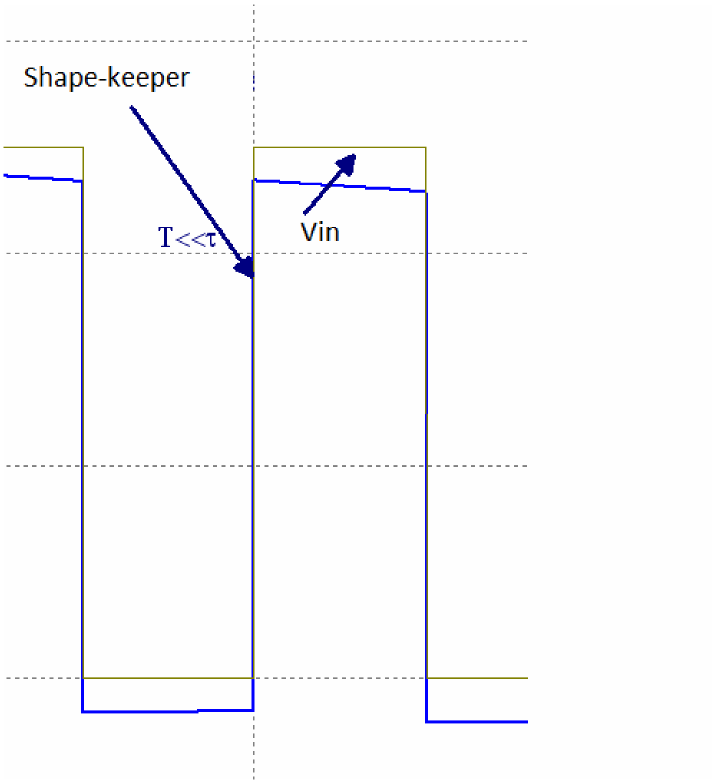
In this case the Bode-diagram contains a phase going upward until the break point, so the connection behaves as a high-pass filter.

We can observe the behaviour in the aspect of time interval the strongly differentiated and the shape-keeper transmission, but the relation of T and τ is the opposite of the integrating units' relations.

In case of differentiating unit the transmission is strongly differentiated if T>>τ



In case of differentiating unit the transmission is shape-keeper if T<<τ



MEASUREMENTS

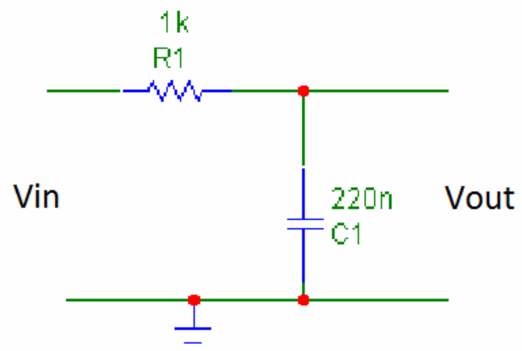
1. Examining the integrating unit

1.1. Examination in the aspect of time interval

We use the square signal of the function generator for our measurements. We can use bigger signals, for example 5V signal amplitude.

Before starting the measurements, let us check the credibility of the time scale of the oscilloscope. Set for example 100Hz signal frequency, its period time is 10ms. We can make the authentication with other signal frequencies! We measure the circuits' output signals directly connected to the oscillioscope!

Then, let us make the following connection using a couple hundred Ω for R and a couple hundred nF for C.



Calculate τ time constant and f_0 cutting frequency:

$$f_0=\omega/2\pi=1/(2\pi RC),\ \tau=1/\omega \qquad (7)$$

Measure τ time constant accoording to figure No 3.

If t=τ, then $V_{out}=V_0^{-1}=V_0/e$.

The decrease of the e order part of the signal means the period when the signal decreases to 37% of V_0 with good approximation.

We can get an evaluable signal form if we set the frequency of the inducing square signal to around 1/3 f_0.

If we did the measurement correctly, then the measured and calculated τ values are equal. A small difference can be caused by that the R and C elements' values can differ from the nominal values within the tolerance.

In the following, $V_{in}=10V_{pp}$ should be constant, and draw the $V_{out}(t)$ function at $1/10*f_0$, f_0 and $10*f_0$ frequency.

1.2. Examination in the aspect of frequency interval

Connect the sinusoidal signal of the function generator to the previously connected circuit and make the following measurements accoording to the following chart.

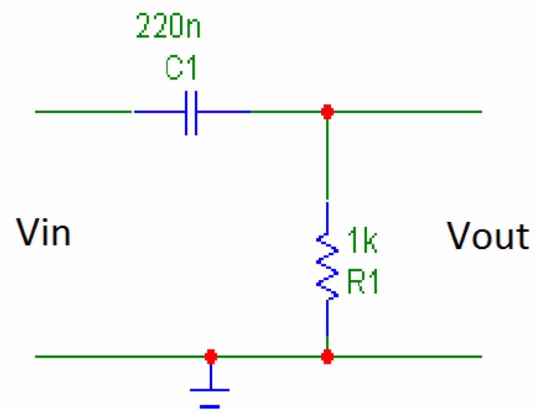
Use AC mode and dB scale of the DMM!

f	V_{in}(dB)	V_{out}(dB)	Au(V_{out}-V_{in})(dB)
50			
100			
400			
500			
600			
f_0			
800	14		
900			
1k			
2k			
5k			
15k			
20k			

Draw the Au(f) transmission function. From the graph, define the steepness after f_0 frequency interval in dB/dec dimension.

2. Examining the differentiating unit

Make the following connection with the previous R C units.



2.1 Examination in the aspect of time interval

We use the square signal of the function generator for our measurements. We can use big signals, for example 5V amplitude.

Draw the signals at f_0, $10f_0$, $100f_0$, $f_0/10$ repeating frequency. Compare the signal transmission of the differentiating and integrating unit! When is the differentiating unit shape-keeper, and when is it strongly differentiating?

2.2 Examination in the aspect of frequency interval

Connect sinusoidal signal to the input, and make the examination in the aspect of frequency interval accoording to the following chart. Represent the transmission function. Define the steepness in the increasing interval in dB/dec dimension.

f	V_{in}(dB)	V_{out}(dB)	Au(V_{out}-V_{in})(dB)
50			
100			
400			
500			
600			
f_0			
800	14		
900			
1k			
2k			
5k			
15k			
20k			

3. Measuring the inductivity of an inductor

The simplest way to determine the unknown inductivity of an inductor is to make a resonant circuit and determine its resonancy frequency, f_r.

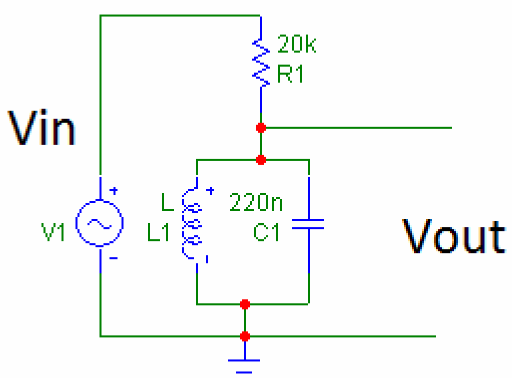


Figure No 4

Remember:

$$f_r=1/2\pi\sqrt{LC} \qquad (8)$$

so, after knowing C and measuring f_r we can calculate L. Make the connection on figure No 4 with R=20kΩ and C=220nF values. Induce the connection with approximately $10V_{pp}$ sinusoidal signal, connect the output to the oscilloscope and stop the time base. Find the frequency where the output amplitude is the maximum by adjusting the generator's frequency.

At this frequency, decrease the generator's amplitude so that the output is not bigger than $100mV_{pp}$. (To avoid the iron core's saturation.)

Find $Vout_{max}$ with careful adjustments again.

This is the resonance frequency f_r.

3.1 Determine the unknown inductivity with the help of f_r and equation (8)

$$L_{measured=}$$

3.2 Knowing the measured inductivity value and the number of rounds in the inductor (n=200) determine the inductivity factor of the iron (A_L).

$$A_L=L/n^2$$

3.3 As a check, calculate the value of the unknown inductivity with the inductivity factor (A_L) given in the guide and the known number of rounds.

$$A_L=1350nH$$

$$n=200$$

$L_{calculated}$=

3.4 Determine and represent the resonancy curve in the frequency interval of $f_r/10$ and f_r.

f	V_{in}(dB)	V_{out}(dB)	Au(V_{out}-V_{in})(dB)
100			
200			
300			
400			
500			
600			
700			
800			
900			
1k			
1.1k			
1.2k			
1.3k			
1.4k			
1.5k			
1.6k			
1.7k			
1.8k			
1.9k			
2k			
3k			
4k			
5k			

Determine the 3dB points. The frequency interval between these points is Δf. The quality factor of the circuit: $Q=f_r/\Delta f$. Determine Q quality factor!

4. LC filter

Cauer filter containing an inductor lengthwise and a capacitor in the cross branch is low-filter-like. Its cut is steeper than the cut of RC filter, but it has a lower enhancement. Make the connection in figure No 5 using C=220nF (or 330 nF). Connecting sinusoidal signal to the input, determine and represent the frequency dependency of the output voltage in the interval of 100Hz...5kHz. Measure the output signal with the dB scale of the DMM.

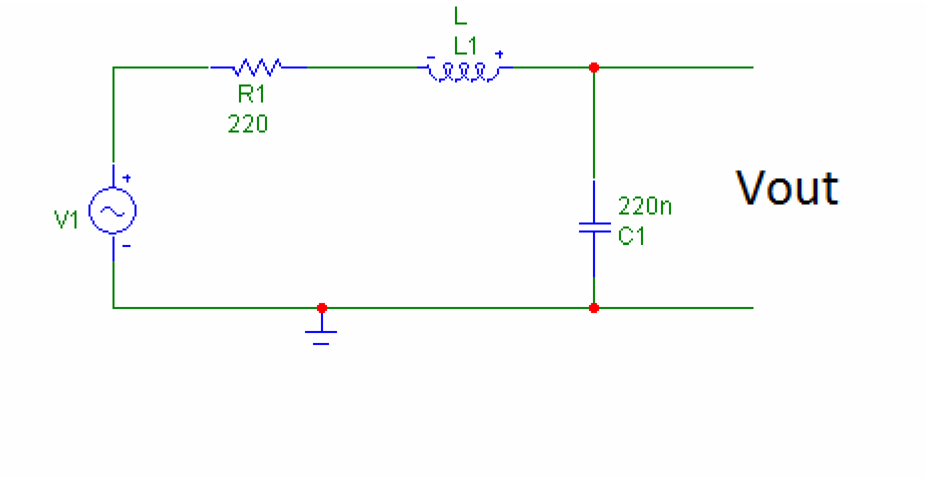


Figure No 5

f	V_{in}(dB)	V_{out}(dB)	Au(V_{out}-V_{in})(dB)
100			
200			
300			
400			
500			

600			
700			
800			
900			
1k			
1.1k			
1.2k			
1.3k			
1.4k			
1.5k			
1.6k			
1.7k			
1.8k			
1.9k			
2k			
3k			
30k			
100k			