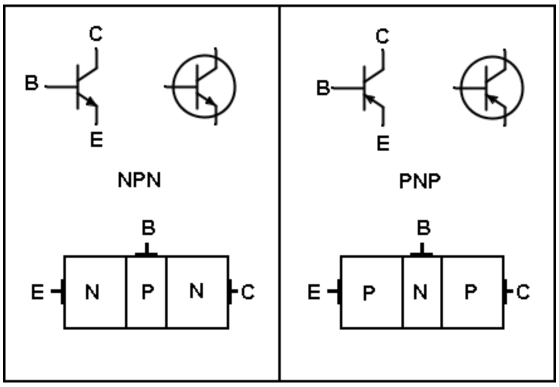
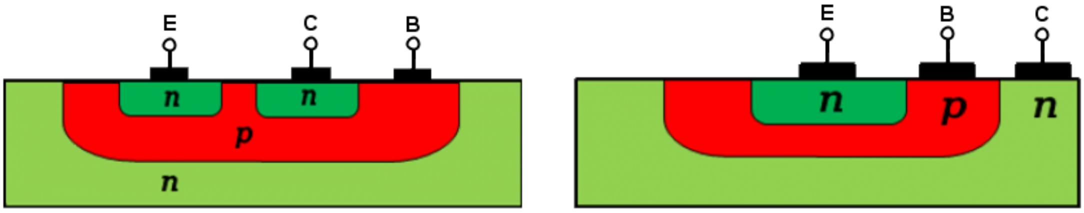
1. Bipolar junction transistor

1.1. Intro

1.1.1. Symbols, structure



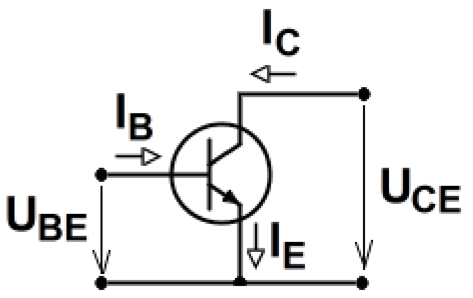
1.: Symbols and simplified structure (order of layers) (left: NPN, right: PNP)



2.: Left: Planar structure NPN. Right: Lateral stucture NPN transistor.

The figures show that the emitter and collector layers are of different size and geometry. They also of different doping concentration. Therefore the E and C are not wholly interchangeable in circuits (the transistor parameters will suffer if doing so).

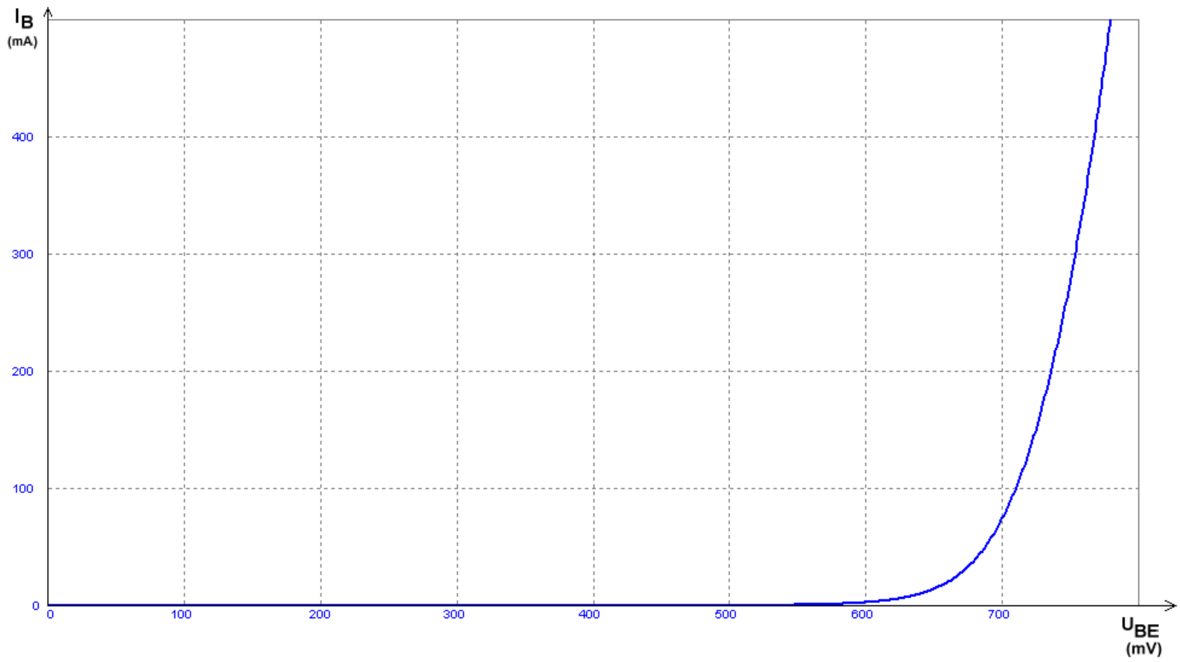
1.1.2. Characteristics



3.: Common Emitter (CE) model

Remember: in practice, do not connect a voltage source directly parallel to BE junction (or to CB junction), without resistor or current limit.

Input characteristic of CE model:



4.: V_{BE}-I_B characteristics, forward region, fix temperature

Remember also that the figures here are just examples. The actual transistors' parameters may slightly vary.

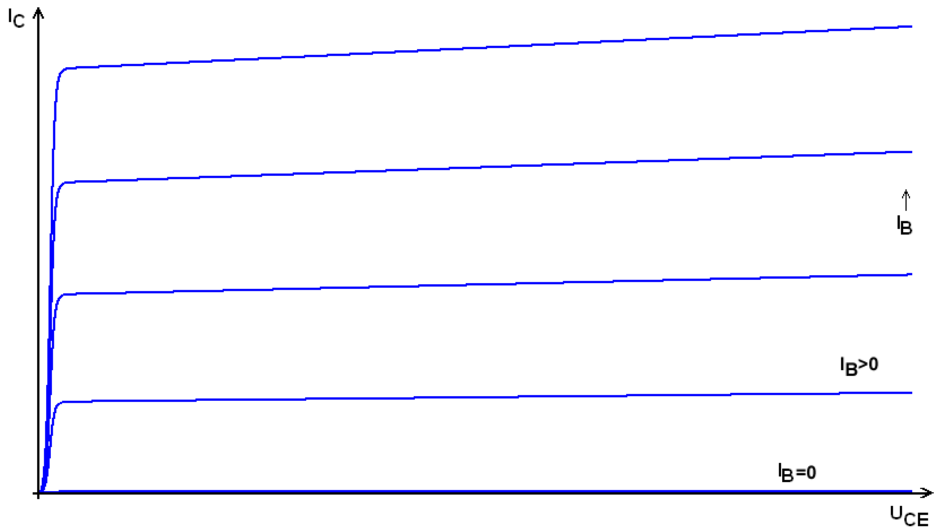
As the BE junction is forward biased, the diode equation can be used here:

$$I_B = I_{B0} \cdot \left(e^{\frac{V_{BE}}{V_T}} - 1 \right)$$

where I_{B0} is the reverse current (or drift current) (very very small, typically pA magnitude) and V_T is the thermal voltage (26mV at room temperature), both temperature dependent.

As with diodes, the characteristic curve is temperature dependent. At higher temperature, the function values (I_B) are multiplied. Looking at the curve, it looks like as if the curve is moving to the left (except that it still crosses the origin). At a fix current, it looks as if the curve moves about -2mV per Kelvin, ie. it moves to the left with increasing temperature. Therefore there is a risk of thermal runaway and thus the warning to not connect a voltage source (without current limit) parallel to the forward biased PN junction.

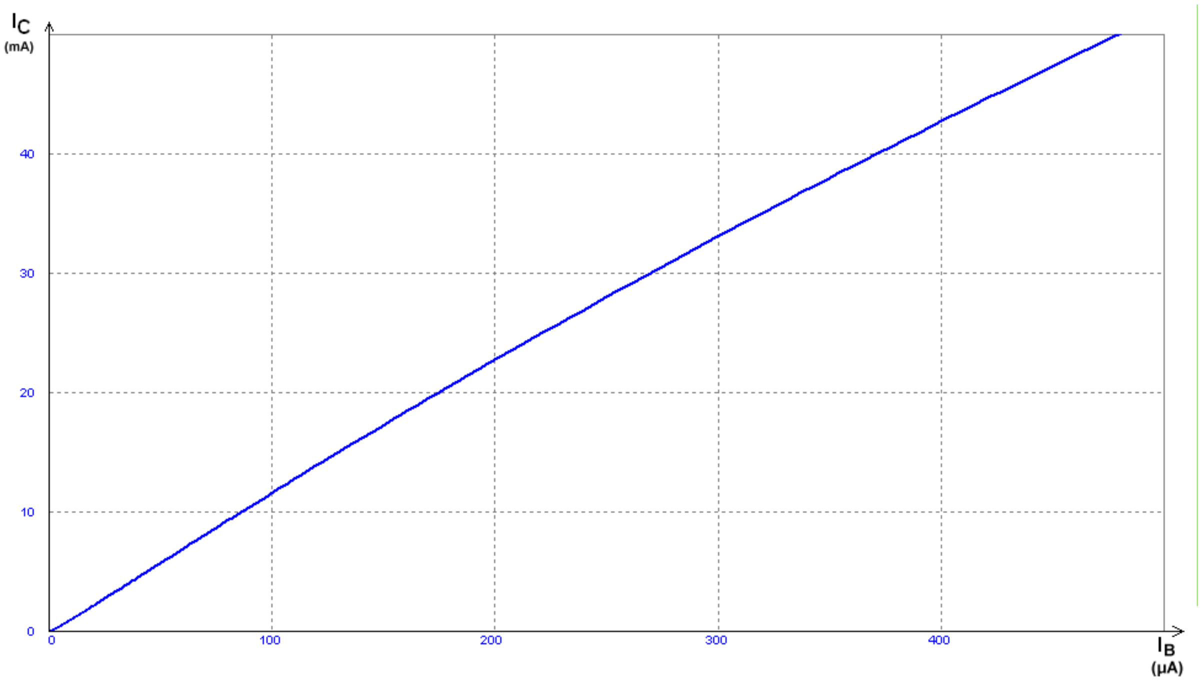
CE output characteristic



5.: V_{CE}-I_C curve

Here we get different V_{CE}-I_C curves for different base currents. I_C is close to zero (see the drift current of the diode) when I_B is zero.
If VCE is sufficiently large (usually after a few hundred mV or a few V), the curve becomes relatively flat, here it behaves as a current generator. In this section we can use the transistor as current generator or as amplifier.

I_C-I_B characteristic (current transfer char.)



6.: I_C-I_B curve

If we have forward bias on V_{BE} and V_{CE} is large enough (to be in the current generator section), then I_C and I_B are approximately linearly proportional:

$$I_C = B \cdot I_B$$

Don't forget that we need a power supply to get this! The transistor can not make higher output current out of air.

The B (capital beta) current gain is ideally constant. For modern low current (typ. <1A) transistors the value of B is between about 100 to 600 (meaning it has a very large tolerance, ie. manufacturing uncertainty). For high current transistors at large I_C and also for very old transistors the B can be much lower, a few ten or less. The curve thus "flattens out" at higher currents.

Node law says:

$$I_C + I_B = I_E$$

therefore

$$B \cdot I_B + I_B = I_E$$
$$(B+1) \cdot I_B = I_E$$

Reorganise and introduce constant A (alpha), which is ratio of I_C and I_E:

$$(B+1) \cdot \frac{I_C}{B} = I_E$$
$$\frac{B+1}{B} I_C = I_E$$
$$I_C = \frac{B}{B+1} I_E$$
$$I_C = A \cdot I_E$$
$$A = \frac{B}{B+1}$$

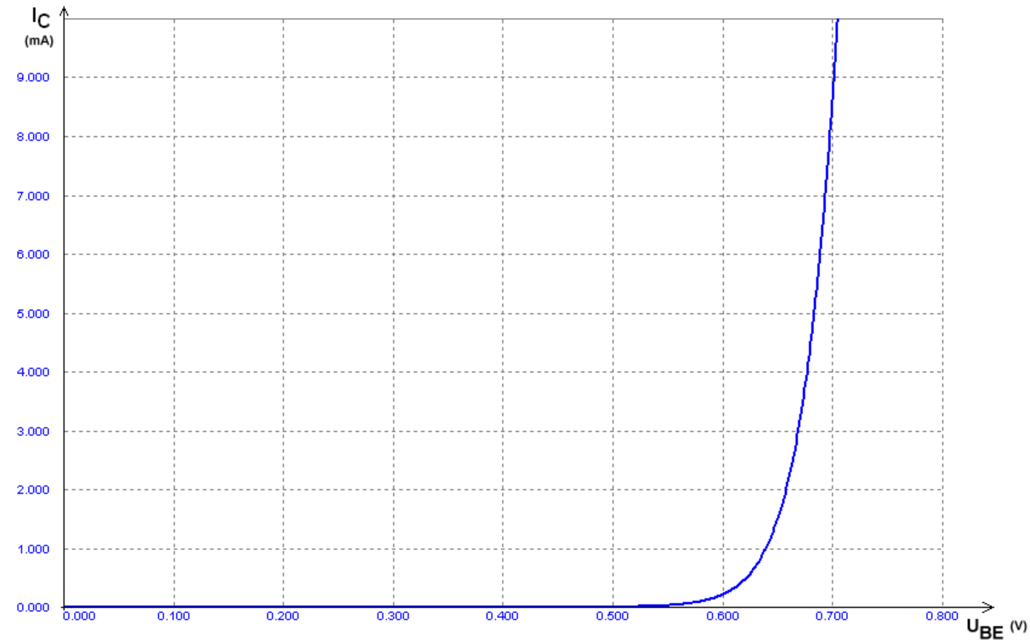
If B>100, then A>0.99, thus we can safely say that I_E=I_C. (Note: at B=100, A=0.99. At B=200, A=0.995. At B=400, A=0.9975. Thus large variations in beta result in negligible change in alpha. Therefore in this regard, we can safely ignore the tolerance of B as long as it's large enough. It requires a carefully designed circuit though, where the output depends on A instead of B.)

V_{BE}-I_C transfer characteristic

With proper voltage applied on both VBE and VCE, we can work the previous characteristics together to get the transfer curve. As I_C=BI_B, it will look similar to the input curve:

$$I_C = I_{C0} \cdot e^{\frac{U_{BE}}{U_T}}$$

(Note the -1 is missing, as I_C will have the I_{C0} drift current even when I_B is zero.)
Remember that this equation is only true as long as VCE is large enough (in the saturated (current generator) section).

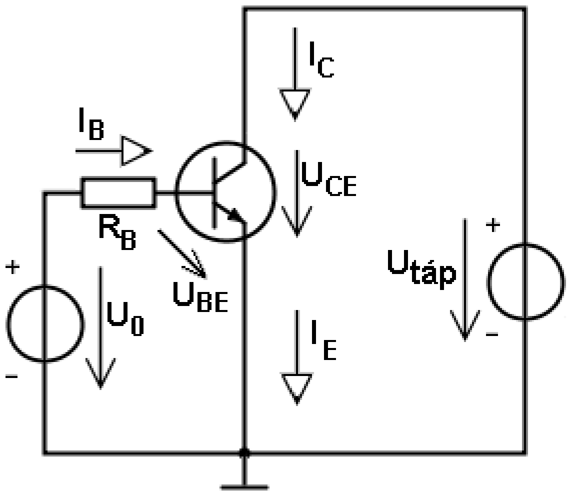


7. V_{BE}-I_C transfer curve

1.2. *Setting the operating point*

For several reasons it is advisable to include a resistor connected to the BE junction. It can either be connected from base or from emitter.

1.2.1. Base resistor method



8. Base resistor / base current setting

If V_0 is larger than the forward voltage, we can assume V_{BE}=0.7V.
Then

$$V_{R_B} = V_0 - V_{BE} = V_0 - 0.7V$$

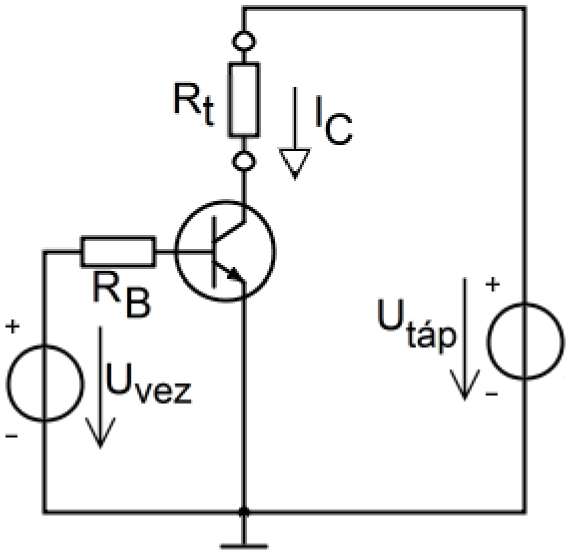
$$I_B = \frac{V_{R_B}}{R_B} = \frac{V_0 - 0.7V}{R_B}$$

$$I_C = B \cdot I_B$$

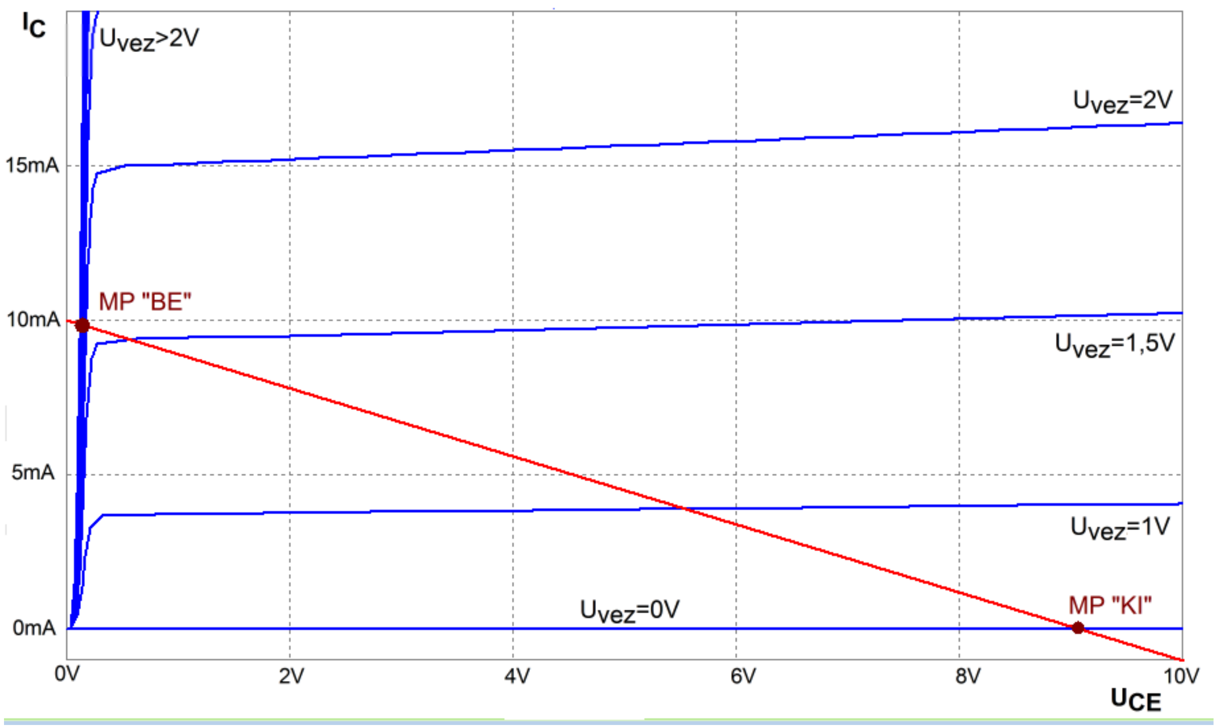
$$I_E = A \cdot I_C \approx I_C$$

Here the IC will depend on B. That is usually not acceptable, as B has a very large uncertainty. This method is thus not used in current generators or amplifiers generally. It can be used in switching mode.

Base resistor method in switching mode



9. Switching mode (Rt is the load)



10. Switching mode. Red curve is the resistor's characteristic. (KI=On, BE=Off, MP=Operating Point)

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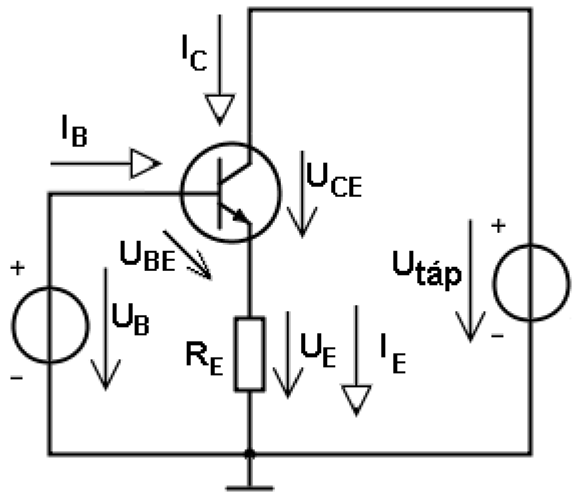
1.2.2. Emitter resistor method



11. Operating point setup with emitter resistor (1)

$$V_{BE} = 0.7V$$
$$V_E = V_B - V_{BE}$$
$$V_{CE} = V_{táp} - V_E$$
$$I_E = \frac{V_E}{R_E}$$
$$\text{if } B >> 1 \text{ (B>100)} \Rightarrow I_B << I_C \Rightarrow I_C \approx I_E$$
$$I_B = \frac{I_C}{B}$$

Here I_C doesn't depend on B, only on the uncertainty of V_{BE} (much better) (and on A, but that is really negligible usually). Here the I_B will depend on B, but that is usually not a problem (the output quantity is usually I_C or related to it).

The larger V_E and R_E, the more precisely I_C can be set, the lower the effect of the uncertainty of the forward voltage.

Example

Let's use these values for an example calculation:
V_B=4.7V ; R_E=2kΩ, V_{supply}=10V (only important thing now is V_{supply}>V_B).

For example a BC182 transistor datasheet says V_{BE} is between 0.55V and 0.7V when I_C=2mA.

First suppose that the given maximum, V_{BE1}=0.7V will be true value.

$$V_B = 4,7V$$
$$V_{BE1} = 0,7V$$
$$V_{E1} = V_B - V_{BE1} = 4V$$
$$I_{E1} = \frac{V_{E1}}{R_E} = \frac{4V}{2k\Omega} = 2mA$$

Now what if VBE is different actually? Let's use now the minimum given value: V_{BE2}=0,55V.

$$V_B = 4,7V$$
$$V_{BE2} = 0,55V$$
$$V_{E2} = V_B - V_{BE2} = 4,15V$$
$$I_{E2} = \frac{V_{E2}}{R_E} = \frac{4,15V}{2k\Omega} = 2,075mA$$

The relative difference between the two calculated emitter currents:

$$I_{E1} = 2mA$$
$$I_{E2} = 2,075mA$$
$$\frac{\Delta I_E}{I_{E2}} = \frac{I_{E2} - I_{E1}}{I_{E2}} = \frac{0,075mA}{2mA} = 0,0375 = 3,75\%$$

Thus 150mV difference in the estimation of VBE (21..27%) will result in 3.75% change in the emitter current. If we used twice the VB and twice the RE, then IE would be the same, but the uncertainty of it only half.

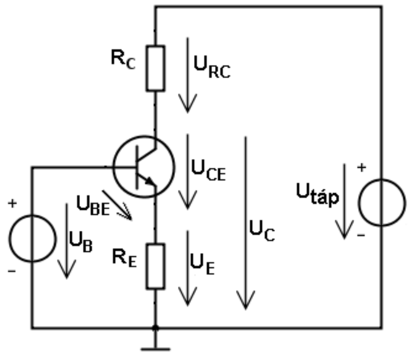
This is why the estimation of VBE to be 0.6V ... 0.7V is usually acceptable.

The emitter resistor also acts a negative feedback.

Suppose for example, that the transistor's temperature increases. If our V_{BE} voltage is constant, then this would result in increased I_C and I_E (as the characteristic curve shifts left). The increased I_C would increase temperature even further, thus getting a positive feedback, a thermal runaway which would destroy the transistor.
If we have R_E in the circuit, then increased I_E results in increased V_E. But $V_{BE}=V_B-V_E$, so with constant V_B, V_{BE} will decrease. But if V_{BE} decreases, then - according to the transfer characteristic curve - I_C and I_E will decrease. We started from I_C and I_E increasing, which then lead to an effect of them decreasing. The two effects try to cancel each other out. (Obviously decreasing I_C would lead to increase.)
Therefore we have a negative feedback. This tries to keep I_C and I_E at near constant value.

This effect works regardless of the reason for change of I_C. It could be the change of R_C (R_L) in a current generator. It could be from AC input signal on the base of the amplifier as well, which means that if we have a common emitter amplifier with R_E but without C_E, then the negative feedback will make the output AC signal very small, ie. it will decrease the voltage gain (compared to the original circuit with C_E).

Current generator



12.:Model of current generator

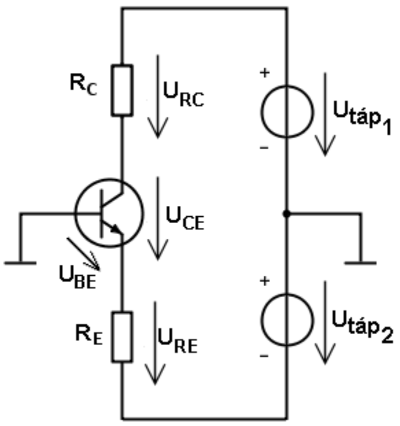
Now we see that the transistor can be used as a current generator if used in the "flat" section of the output characteristic. The emitter resistor has two roles here: first, it allows setting up I_C independently of value of (uncertainty of) B and greatly decreases the effect of uncertainty of V_{BE}. Second, it provides the negative feedback which tries to keep I_C from changing, which adds to the already existing effect of being a current generator - in effect, it tries to make the output curve even flatter.

The collector potential is calculated as:

$$V_C = V_{supply} - I_C R_C$$

Remember, the ⊥ symbol here denotes the reference zero for node potentials. Symbols with one letter index, such as V_C, indicate node potentials. Symbols with two indices indicate voltages between those points, like V_{CE}. So VC could be written also as V_{C0}.

Using double power supply



13. Using double power supply

In this case we use two voltage generators to provide a + and a - voltage relative to the zero (which is now between the two generators and indicated by the ⊥ symbol).

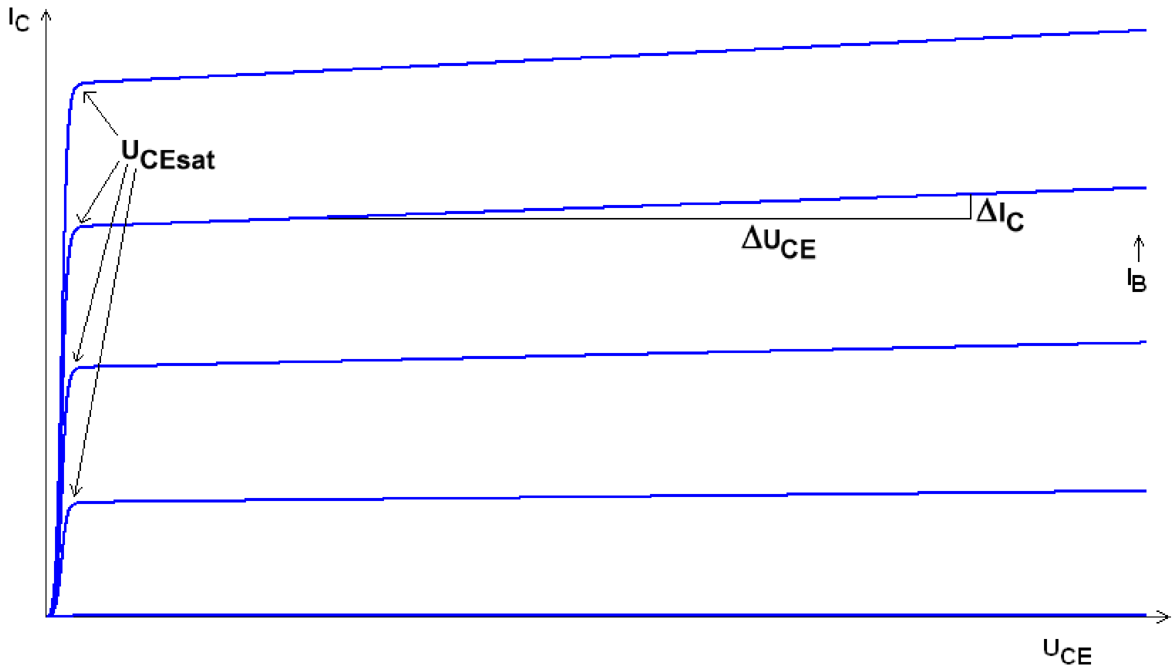
In this case, V_E and V_{RE} are different, as the bottom of RE is at negative potential, not zero.

If for example $V_{supply1}$=10V, $V_{supply2}$=5V, R_E=R_C=1kΩ:

$$V_E = 0 - V_{BE} = -0,7V$$
$$V_{RE} = V_E - (-V_{supply2}) = -0,7 - (-5) = 4,3V$$
$$I_E = \frac{V_{RE}}{R_E} = \frac{4,3V}{1k\Omega} = 4,3mA$$
$$I_C \approx I_E$$
$$V_{RC} = I_C R_C = 4,3V$$
$$V_C = V_{táp1} - V_{RC} = 10V - 4,3V = 5,7V$$

For R_C and V_C, it behaves similarly to the previous circuit.

1.3. Current generator



14. V_{CEsat} and r_{CE} explanation

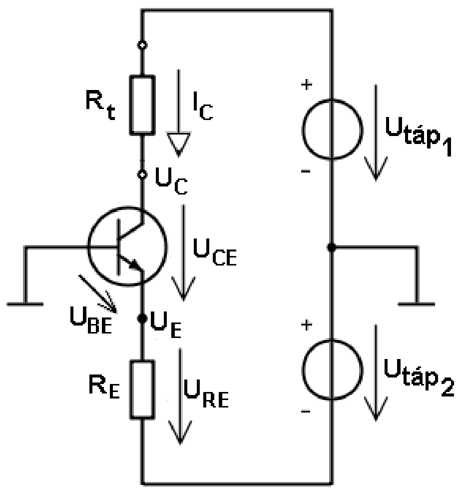
The quality of the current generator (how constant is I_C) depends on the "flatness" (slope) of the VCE-IC curve, ie. the dynamic output resistance of the transistor.

$$r_{CE} = \frac{dV_{CE}}{dI_C} \approx \frac{\Delta V_{CE}}{\Delta I_C}$$

In the right side of the curve this is seen to be very large (ie very flat). This can be from 30kΩ to 100kΩ or more. (Remember an ideal current generator has infinite internal resistance.)
As mentioned already, the RE as negative feedback makes the circuit's output resistance even greater than that provided by the transistor (r_{CE}) itself.

V_{CEsat} is the minimum voltage needed to reach the saturation (current generator mode).
We can see this value is greater if I_C is greater. For low currents (few mA) this is usually a few hundred mV. For higher currents it could be volts.

Current generator with double supply

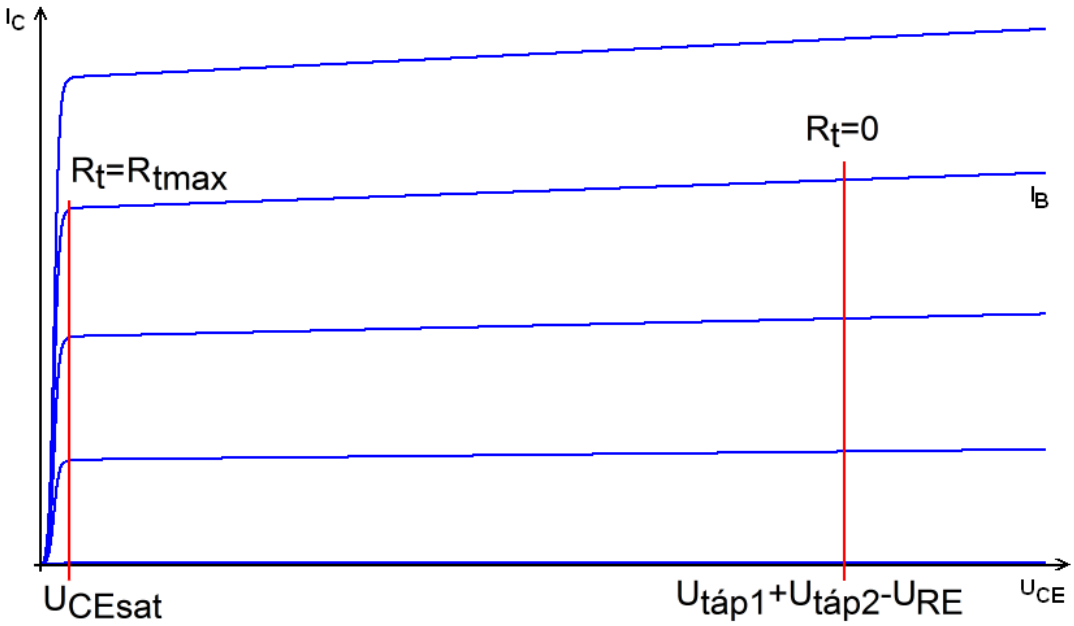


15.

$$U_E = 0 - U_{BE} = -0,7V$$
$$U_{RE} = U_E - (-U_{táp2})$$
$$I_E = \frac{U_{RE}}{R_E}$$
$$I_C \approx I_E \text{ (B nagy)}$$

Note: when writing V_{supply} ($U_{táp}$ in Hungarian) next to a voltage arrow (next to a voltage generator), it is considered a voltage and is positive. But when used as a potential, it can be negative: $V_{supply2}$ as potential (when written next to a node, in this case below the R_E) will be negative, because its related generator is connected below the zero potential.

Value of $V_{supply2}$ is chosen similarly as V_B in the one supply version. It should be greater than V_{BE} (0.7V), possibly by several times, but not too much.



16.

What is the limit for the load resistor?

The ideal generator can be short circuited. Then the voltage on the load is zero. This works here, thus $R_{Lmin}=0$.

In this case

$$V_{CE\max} = V_{supply1} + V_{supply2} - V_{RE} = V_{supply1} + V_{supply2} - I_E \cdot R_E$$

(note: $V_{supply1}$ and $V_{supply2}$ are voltages here, all positive)

When increasing R_L, the current will only very slightly decrease due to the finite output resistance. But after reaching R_{Lmax}, the I_C will decrease strongly.

This happens when the voltage on the R_L becomes so great that there remains no sufficient voltage to bring the transistor V_{CE} into the saturated region (which requires minimun of V_{CEsat} voltage). When further increasing R_L, the potential V_C becomes lower than V_B and thus the C-B junction is forward biased, and current also flows from base to collector, subtracting from I_C.

If V_{CEsat} is known at the given I_C :

$$V_{RL\max} = V_{supply1} + V_{supply2} - V_{CEsat} - V_{RE} = V_{supply1} + V_{supply2} - V_{CEsat} - I_E \cdot R_E$$

$$R_{L\max} = \frac{V_{RL\max}}{I_C} = \frac{V_{supply1} + V_{supply2} - V_{CEsat} - V_{RE}}{I_C}$$

Thus R_{Lmax} depends on power supply voltage and the saturation voltage.

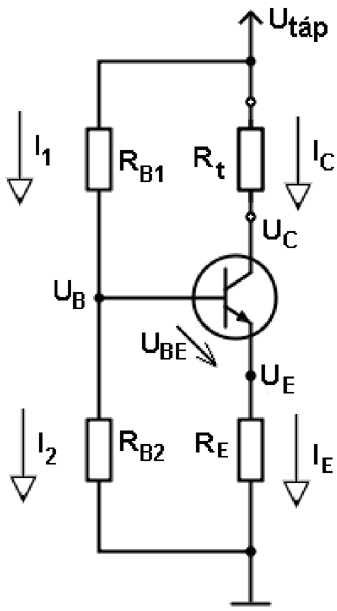
In the double power supply circuit, V_E=-0,7V and thus:

$$V_{RL\max} = V_{supply1} + 0,7V - V_{CEsat}$$

$$R_{L\max} = \frac{V_{RC\max}}{I_C} = \frac{V_{supply1} + 0,7V - V_{CEsat}}{I_C}$$

So R_{Lmax} only depends on $V_{supply1}$, not on $V_{supply2}$.

1.3.1. Current generator with one power supply



17. Current generator

In practice, using two generators is expensive (cost, size, complexity). Therefore we replace $V_{supply2}$ with a voltage divider. The simplest version is made from two resistors. Another version would use a resistor and a Zener-diode, this way it becomes almost independent from power supply value.

From now we use the up arrow to symbol the power supply potential, instead of showing the generator fully connected. This makes for easier overview of our circuits.

In this circuit, $V_E=V_{RE}$ and $V_B=V_{RB2}$.

$$V_{RL\max} = V_{supply} - V_{CEsat} - V_E = V_{supply} - V_{CEsat} - I_E \cdot R_E$$

$$R_{L\max} = \frac{V_{RL\max}}{I_C} = \frac{V_{supply} - V_{CEsat} - V_E}{I_C}$$

When designing, value of V_E can be chosen somewhat freely. Larger V_E means more precise setting of I_C but also lower R_{Lmax} and also higher power dissipation (counts if current is larger than a few hundred mA).

Values of R_{B1} and R_{B2} can be choosen also somewhat freely, as it's their ratio that counts. We have to make sure $I_1>I_B$ (as $I_2=I_1-I_B$ and of course I_2 should be positive).

$$I_1 = \text{choose } (I_1 > I_B)$$
$$I_2 = I_1 - I_B$$
$$R_1 = \frac{V_{supply} - V_B}{I_1}$$
$$R_2 = \frac{V_B}{I_2}$$

A usual practice for choosing I_1 is
let $I_1 = 10 \cdot I_B$
then $I_2 = I_1 - I_B = 9 \cdot I_B$

When calculating an existing circuit, it is complicated to precisely calculate V_B and the result would still depend on B. Therefore we assume that the designer followed the above guideline and thus $I_1>>I_B$ and we estimate V_B from voltage divider:

$$V_B \approx V_{supply} \frac{R_{B2}}{R_{B1} + R_{B2}}$$

(*Don't forget to check the validity of this estimation at the end.*)

From here:

$$V_E = V_B - 0,7V$$
$$I_E = \frac{V_E}{R_E}$$
$$I_C \approx I_E$$

Check:

$$I_B = \frac{I_C}{B}$$
$$I_1 = \frac{V_{supply} - V_B}{R_{B1}}$$
$$I_1 > 10 \cdot I_B \text{ ??}$$

1.3.2. Output resistance of current generator

As mentioned previously, the output resistance of the current generator depends on r_{CE} and on the feedback effect of R_E. To measure it:

$$r_{out} = \frac{dV_{out}}{dI_{out}} = \frac{dV_{RL}}{dI_C} \approx \frac{\Delta V_{RL}}{\Delta I_C}$$

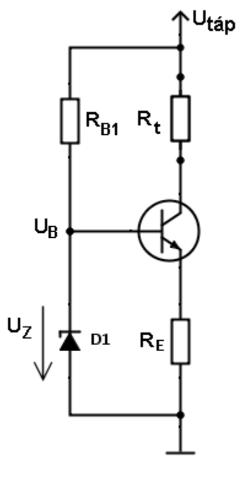
This can tell us how much I_C changes if the load changes.
For example if RL takes two possible extreme values R_{L1} and R_{L2}:
I_C=1mA, r_{out}=500kΩ, R_{L1}=0, R_{L2}=1kΩ.

Then (as long as $R_{L2}<R_{Lmax}$) the change in I_C:

$$\Delta I_C = \frac{\Delta V_{RL}}{r_{out}} = \frac{R_L \Delta I_C + I_C \Delta R_L}{r_{out}} \approx \frac{I_C \Delta R_L}{r_{out}} = \frac{I_C (R_{L2} - R_{L1})}{r_{out}} = \frac{1mA \cdot 1k\Omega}{500k\Omega} = 2\mu A$$

1.3.3. Current generator with Z-diode

In this case the Zener-diode tries to keep V_B somewhat constant even if V_{supply} changes (as long as $V_{supply}>V_Z$ of course). This can be useful for example if the supply is fluctuating (like from a rectifier output) or is slowly decreasing (battery depleting).



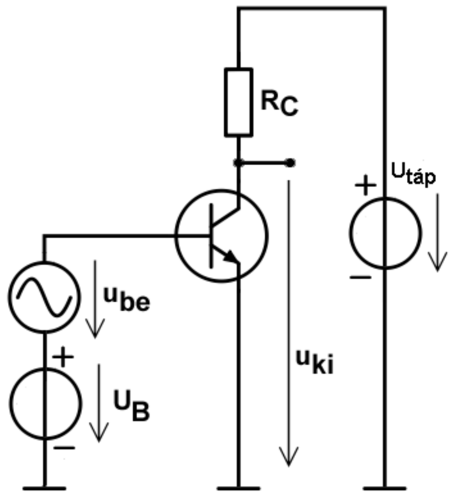
18. using Z-diode base divider

Make sure that the Z-diode gets minimum a few mA of current to properly work. (This is usually greater than what an R-divider would need).

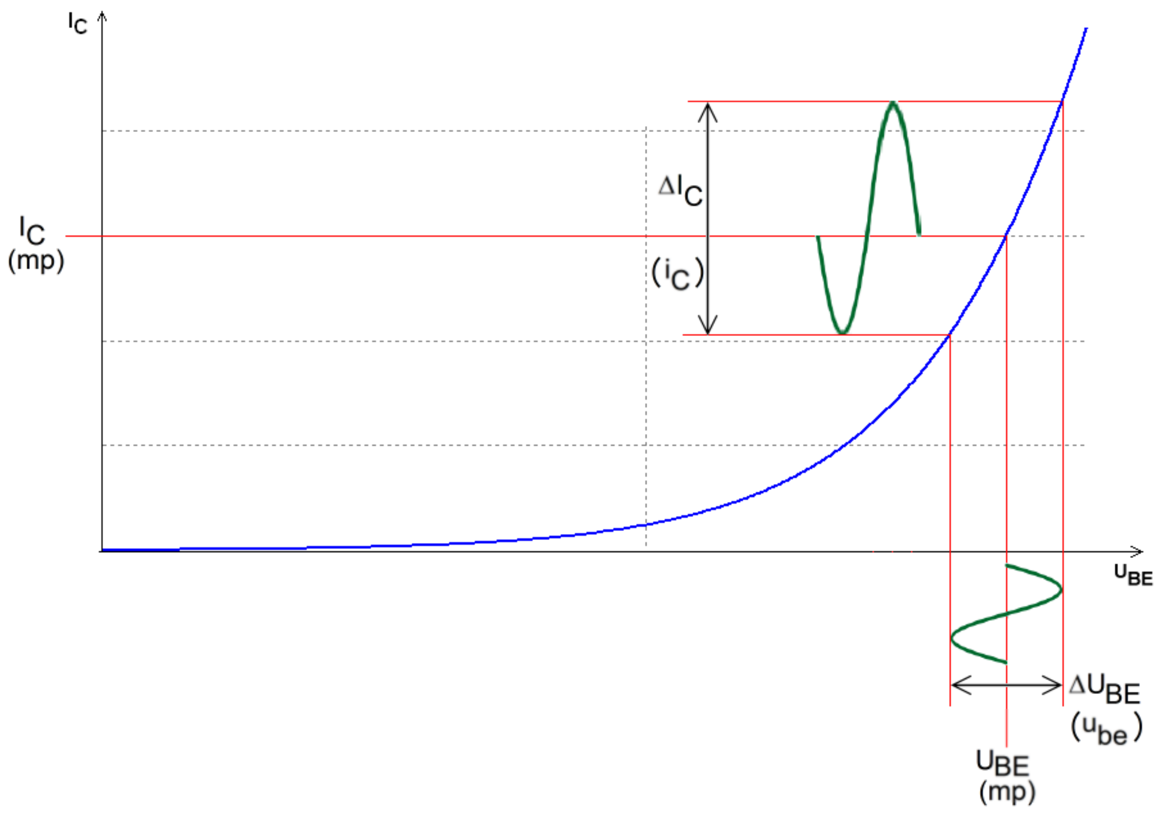
1.4. Simple amplifier circuits

1.4.1. Common Emitter (CE) amplifier

The amplification process



19. CE model



20.

Definition of gains (amplifications) for any amplifier:

$$A_V = \frac{v_{out}}{v_{in}}; \quad A_I = \frac{i_{out}}{i_{in}}; \quad A_P = \frac{p_{out}}{p_{in}}$$

*Note that the **definition** is a formula that can be used to measure the quantity or to derive the actual formula for the given circuit.*

We are going to find AV gain for the "*small signal approximation*". This means that the AC input signal (v_{in}) is much smaller than the V_{BE} DC bias voltage. Thus the vin projected onto the input or transfer characteristic curves will occupy only a short section, which will be treated as almost linear. See figure 20.

This way the connection between v_{in} and i_C can be given by the slope of the characteristic curve at the operating point.

The slope (also called transfer conductance, or transconductance in short) is defined generally as

$$g_{21} = \frac{\Delta I_{out}}{\Delta U_{in}} = \frac{i_{out}}{u_{in}}$$

Note: it's denoted by *g* as it is a conductance and thus measured in S (siemens). In Hungarian it is g_m (as slope is *meredekség*) also sometimes used in international literature (where m stands for *mutual*) In German its symbol is S. The lower case g shows that it is a ratio of changes, ie. a dynamic quantity.

This formula for g_{21} is universal, it can be applied to any device which has an input voltage - output current characteristic (such as other types of transistors or tubes).

In a bipolar CE amplifier this leads to:

$$g_{21} = \frac{\Delta I_C}{\Delta V_{BE}} = \frac{i_C}{u_{BE}}$$

Knowing the characteristic curve, we can find the transconductance at any operating point. It is the slope of the tangent line at the operating point, or in other words, the differential of the function at the operating point.

$$g_{21} = \frac{dI_C}{dV_{BE}} = \frac{d(I_{C0} e^{\frac{V_{BE}}{V_T}})}{dV_{BE}} = \frac{1}{V_T} I_{C0} e^{\frac{V_{BE}}{V_T}} = \frac{I_C}{V_T} \quad \left[\frac{A}{V} = S\right]$$

where V_T=26mV (thermal voltage) at around 300K. körül) hányadosával. Result of g_{21} is usually in millisiemens.

Knowing g21 and the input signal we can get i_C, from which, knowing R_C we get the output voltage (first approximation).

The collector potential generally:

$$V_C = V_{supply} - V_{RC} = V_{supply} - I_C R_C$$

Here I_C has both a DC and an AC component. Now we only want to know the AC component (denoted by i_C), so the DC components (V_{supply} and I_C) disappear (but the negative sign stays):

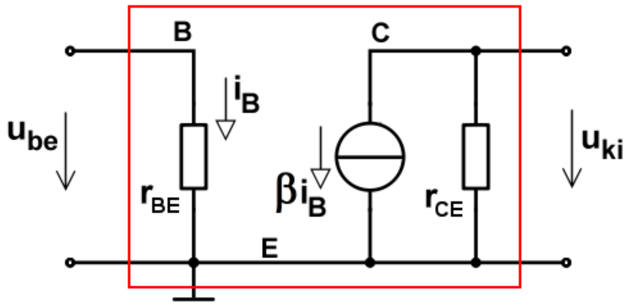
$$v_{out} = v_C = -u_{RC} = -i_C \cdot R_C$$

Knowing that the $v_{in}=v_{BE}$ in this circuit, take the formula for gain and substitute g_{21} into it:

$$A_V = \frac{v_{out}}{v_{in}} = \frac{-i_C R_C}{v_{in}} = -\frac{i_C}{v_{in}} R_C = -\frac{i_C}{v_{BE}} R_C = -g_{21} \cdot R_C$$

Note: This formula, by the way, is a generally true one for similar circuits (if you substitute r_{out} for R_C). Ie. it is true for JFET and MOSFET and tube amplifiers as well in configurations similar to common emitter. It's the actual end formula for g_{21} that will differ.

A more accurate model of the amplifier follows, leading to a more accurate formula.



21. AC small signal model of transistor

The lower case r values are dynamic resistances, ie. ratios of change of voltage and change of current, or in other words, AC parameters.
This model is only true as long as the circuit gets an adequate dc power supply. But that is not shown here, as it is an AC model. We get this by using method of superposition. First turn off the AC sources and calculate all the DC voltages and currents (get the operating point). Then you can turn off the DC supply and turn on the AC input (the signal to be amplified) and calculate the AC values. (The order of these operations matters, as the AC parameters are dependent on the DC operating point.)

The input can be modeled by an impedance, for now simplified as a real resistance. This is modeled by r_{BE} - it is the input dynamic resistance of the transistor, ie change of V_{BE} over change of I_B. This determines the amplifier's input resistance.

As the V_{BE}-I_B characteristic is again coming from the known diode equation, the result will be similar to the derivation of g_{21}.

$$r_{BE} = \frac{dV_{BE}}{dI_B} = \frac{1}{\frac{dI_B}{dV_{BE}}} = \frac{1}{\frac{dI_{B0}(e^{\frac{V_{BE}}{V_T}} - 1)}{dV_{BE}}} = \frac{V_T}{I_B}$$

The output contains a controlled (AC) current generator and an output dynamic resistance (Norton-model). The generator creates beta times the input current.
The output resistance is r_{CE}, already met when discussing the current generator circuit.
It is defined as

$$r_{CE} = \frac{dV_{CE}}{dI_C}$$

Normally it should be several ten or hundred kΩ minimum (in the saturated part of the curve!).
It is greater, if I_C is smaller.

You can find h-parameters in many books and datasheet. These are practically equivalent to the notations here as follows:

$$h_{11e} = r_{BE}$$

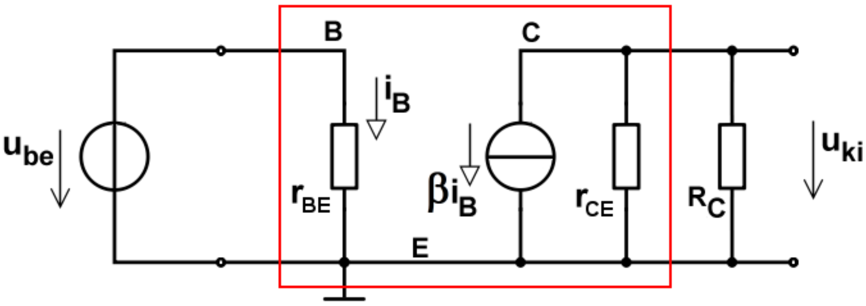
$$h_{21e} = h_{FE} = \beta$$ [1]

$$\frac{1}{h_{22e}} = r_{CE}$$

(Note that h-parameters have conditions for their measurement/definition, but we are not going to discuss that here.)

[1] Theoretically the DC value of beta is capital B, the small signal AC value of beta is lowercase β (defined as $\beta = \frac{dI_C}{dI_B}$). The difference is not important now and the wide range of literature also differs greatly in usage and correctness. (Ie. h-parameters, indicated by the lowercase, should be AC as well, but still h21 is used for DC beta as well usually). We shall treat B and β as equal here.

Let's put R_C into our AC model now. Using superposition, deactivating the DC power supply, it becomes a short circuit to the ground (being a voltage source with zero internal resistance). Thus all resistors that are connected to the power supply, are connected to ground in AC. Thus R_C is here connected between collector and ground in AC, thus parallel with r_{CE}.



22. Unloaded amplifier AC model (no R_L)

$$i_B = \frac{v_{in}}{r_{BE}}$$

$$i_C = \beta \cdot i_B = \beta \frac{v_{in}}{r_{BE}}$$ [2]

$$u_{out} = -i_C \cdot (r_{CE} \,||\, R_C) = \beta \frac{v_{in}}{r_{BE}} (r_{CE} \,||\, R_C)$$

$$A_v = \frac{v_{out}}{v_{in}} = -\frac{\beta}{r_{BE}} (r_{CE} \,||\, R_C)$$

This, at first sight, looks different from what we got earlier. (Unfortunately, many books stop here and give this as result and thus the student will not understand why the circuit is independent of beta).

But if we substitute the formula for r_{BE}:

$$r_{BE} = \frac{V_T}{I_B} = \frac{V_T \beta}{I_C}$$

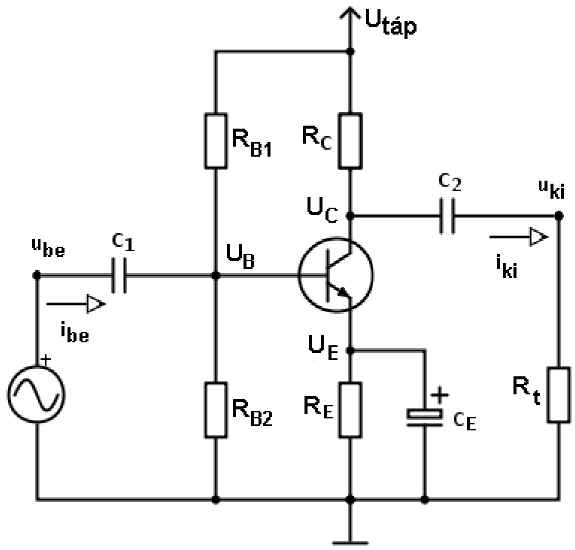
$$A_V = -\frac{\beta}{r_{BE}} (r_{CE} \,||\, R_C) = -\frac{\beta I_B}{V_T} (r_{CE} \,||\, R_C) = -\frac{I_C}{V_T} (r_{CE} \,||\, R_C) = -g_{21} (r_{CE} \,||\, R_C)$$

we get a result that is familier, only now extended by r_{CE}. In practice, r_{CE} is usually much greater than RC, and thus can be neglected, getting back the earlier formula.

This way we can see that the gain does not depend on beta. The value of g_{21} is dependent on I_C. In reality the amplifier is based on the current generator (so as to get stable I_C for a stable gain) and thus we have an R_E which sets up I_C such that B practically doesn't affect it. Thus the gain is relatively stable.

[2] Remember: I'll be using the "parallel" symbol for calculating parallel net resistance. In Hungarian books it is often denoted by an x, but that can be mistaken for multiplication or for cross product.

Common emitter amplifier with one power supply



23. CE amplifier with AC coupling (Rt = RLoad)

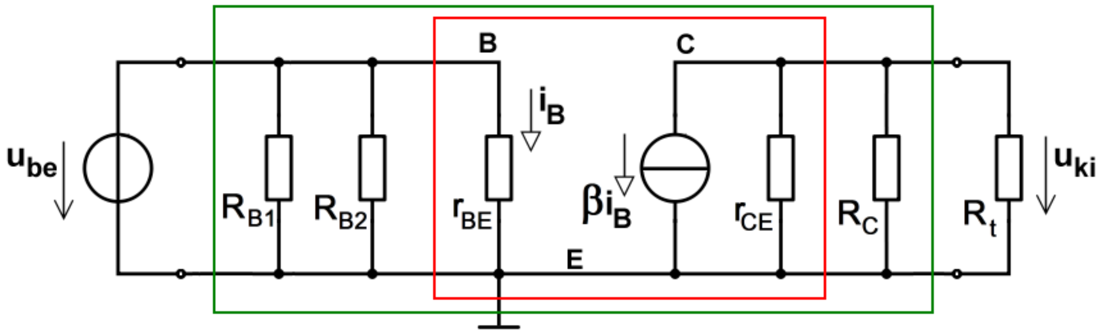
We can see that this circuit is based on the current generator. As mentioned, the gain is dependent on I_C, therefore we want to keep I_C constant (in DC only, of course). Also, V_C determines the max output voltage swing (range), which is also important. V_C of course also depends on I_C.

The capacitors C_1 and C_2 disconnect the input and output in DC. This is required when the two sides have different DC voltage. Eg. in the laboratory, the input is gained from a function generator which has a DC voltage of 0V and very low output resistance; thus connecting it without C_1 would make V_B=0 as well.
The output capacitors makes IC independent of RL and is also important if the load circuitry needs different DC voltage. (In reality we often have several amps connected).

The emitter capacitor (C_E) has another role. If it is not present, then R_E will create a negative feedback - it tries to keep I_C constant. That means it suppresses the AC component i_C, therefore it greatly decreases the gain. But we can't remove R_E, because it sets up the DC I_C current and stabilizes it. Therefore we put in C_E, which short circuits R_E only for purposes of AC signal. Therefore the earlier AC model we presented is still true, if the impedance of C_E is relatively small at the frequency of the input signal. This also means that at lower frequencies, the impedance of C_E is not low enough and thus some negative feedback still occurs. Thus C_E greatly determines the lower limit frequency. C_1 and C_2 also affect it, but C_E has the greatest effect.

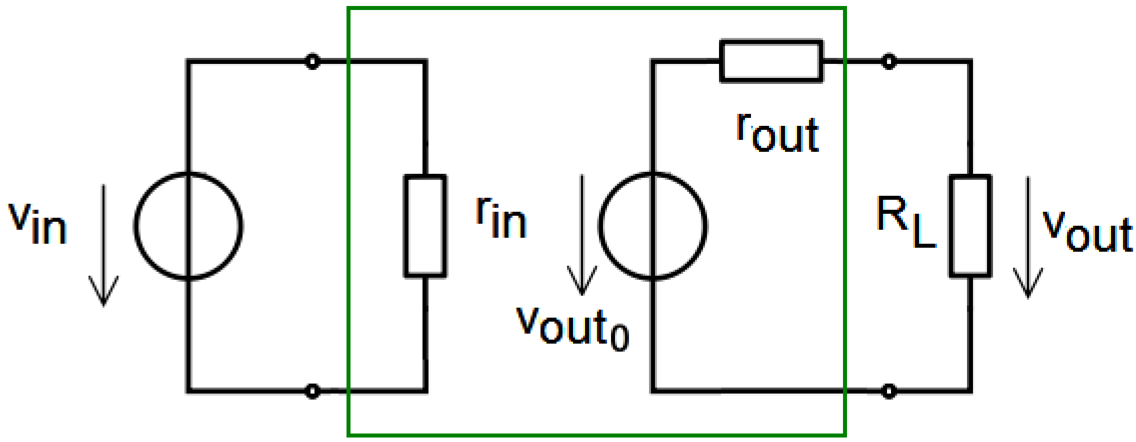
For now, suppose that in operating frequency range all three capacitors can be considered short circuits (zero ohm).

Again, using superposition we find that R_{B1} and R_{B2} are both connected between base and ground and so actually in parallel in AC. We can now draw the full AC model:



24. AC small signal model of CE amplifier. Red: transistor model, green: amplifier circuit model

Let's take a look at a general model of any amplifier:



25. simplified model of a general amplifier

The amplifier can be modeled by a resistor on the input and a controlled generator at the output. Here I have drawn a Thevenin-model, as it is more well known probably, and is more useful is measurements. But remember that the transistor's model is actually a Norton-model (because of $I_C=BI_B$). Also remember the two models can be changed into each other, so we can use both here.)

Let's find out these parameters for our CE amplifier:

$$r_{in} = R_{B1} \,||\, R_{B2} \,||\, r_{BE}$$
$$r_{out} = r_{CE} \,||\, R_C$$
$$A_V = -g_m \left(r_{CE} \,||\, R_C \,||\, R_L\right) = -g_m \left(r_{out} \,||\, R_t\right)$$
$$A_V \approx -g_m \left(R_C \,||\, R_t\right),\ \text{if}\ R_C \ll r_{CE}$$
$$A_V \approx -g_m R_C,\ \text{if}\ R_C \ll r_{CE}\ \text{and}\ R_L = \infty$$

So r_{CE} and R_C make up the output resistance. Don't forget that the load (R_L) is not part of the amplifier and thus not part of the output resistance. If R_L is not present or very large, then approximate $R_L = \infty$ which simplifies to the already known formula then.

Current gain is only present if there is a finite load:

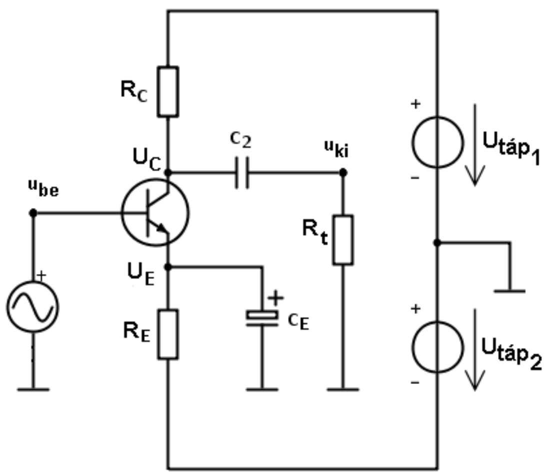
$$A_I = \frac{i_{out}}{i_{in}} = \frac{\frac{v_{out}}{R_L}}{\frac{v_{in}}{r_{in}}} = \frac{v_{out}}{v_{in}} \cdot \frac{r_{in}}{R_L} = A_V \frac{r_{in}}{R_L}$$

Notice that this formula is independent of the actual circuit and so can be used generally for any amplifier where our model of 25 is true.

Power gain:

$$A_P = \frac{p_{out}}{p_{in}} = \frac{v_{out} \cdot i_{out}}{v_{in} \cdot i_{in}} = A_V \cdot A_I$$

CE amplifier with double power supply



26.

The CE amp can also be made with two power supplies. This removes the need for the base resistors. Here we supposed the input generator has DC voltage of zero, thus can be directly connected. Otherwise C_1 capacitor would still be needed! In such a case the transistor's base has to be connected to the ground via a large resistor, to give it the zero volt operating point.

In this case the calculations in DC are similar as the double supplied current generators. AC calculations are similar to the single supplied amp, just R_{B1} and R_{B2} are not present in the equation (except if R_{B2} is connected because of using C_1).

Exercise: CE amplifier

Parameters:
supply: V_0=20V ;
R_{B1}=169kΩ ; R_{B2}=33,2kΩ ; R_C=5,1kΩ ; R_E=2,21kΩ ; R_L=7,15kΩ ;
C_E ; C_1 ; C_2 : very large;
B=400

$$V_B \approx V_0 \frac{R_{B2}}{R_{B1}+R_{B2}} = 3,28V$$
$$V_E = V_B - 0,7V = 2,58V$$
$$I_E = \frac{V_E}{R_E} = \frac{2,58V}{2210\Omega} = 1,17mA$$
$$I_C \approx I_E = 1,17mA$$
$$I_B = \frac{I_C}{B} = \frac{1,17mA}{400} = 2,92\mu A$$
$$V_C = V_0 - I_C R_C = 20V - 1,17mA \cdot 7,15k\Omega = 11,64V$$
$$r_B = \frac{V_T}{I_B} = \frac{26mV}{2,92\mu A} = 8,9k\Omega$$
$$r_{in} = r_B \,||\, R_{B1} \,||\, R_{B2} = 6,74k\Omega$$
$$r_{out} = R_C \,||\, r_{CE} \approx R_C = 7,15k\Omega$$
$$g_{21} = \frac{I_C}{V_T} = \frac{1,17mA}{26mV} = 45mS$$
$$A_V = -g_{21}(r_{out} \,||\, R_t) = -161$$
$$A_V(dB) = 20\lg\left|A_V\right| = 44dB$$
$$A_I = A_V \frac{r_{in}}{R_L} = 152$$

Check for the validity of the starting supposition. (Note: checking for I1 or I2 are equally good.)

$$I_2 = \frac{V_B}{R_{B2}} = \frac{3,28V}{33,2k\Omega} \approx 100\mu A$$
$$I_B = \frac{I_C}{B} = \frac{1,17mA}{400} \approx 3\mu A$$
$$I_2 \geq 10 \cdot I_B$$

This means our supposition is acceptable (if not very good).

Maximum output voltage, maximum gain

The actual potential (ie at any time moment) of the collector can not exceed the positive power supply, or go below the emitter potential (because C_E keeps it constant). Thus the voltage swing (peaks) of the output are limited.

If the input signal times the gain would give larger signal than what the power supply, it will be cut off (made into a square wave).

Let's use a symmetric input signal (ie positive and negative peaks equal), such as a sine wave. Let's look at an unloaded circuit for simplicity.

For maximum output voltage peaks, we have to put VC in middle of its range:

$$V_C \Leftarrow \frac{V_{supply} + V_E + V_{CEsat}}{2} \approx \frac{V_{supply} + V_E}{2}$$

In practice we don't exactly keep this, but generally try to put VC around this value (roughly half the power supply value).

When R_L is present, the collector potential can not reach Vsupply. If eg. $R_C = R_L$ and operating point is V_C=$V_{tsupply}$2 for simplicity, then C_2 (output capacitor) charges up to $V_{supply}/2$. The rest of the voltage (also $V_{supply}/2$) is divided between the resistors evenly and thus V_C can only go up to 3/4 V_{supply}. But in any case, driving the circuit to maximum would often cause too much distortion so we won't use the full range probably.

To find the maximum theoretical gain, suppose RL is infinite (not present) and maximum output voltage range. Take the simplified circuit of figure 19.

$$V_C = \frac{V_{supply}}{2}$$

$$I_C R_C = V_{supply} - V_C = \frac{V_{supply}}{2}$$

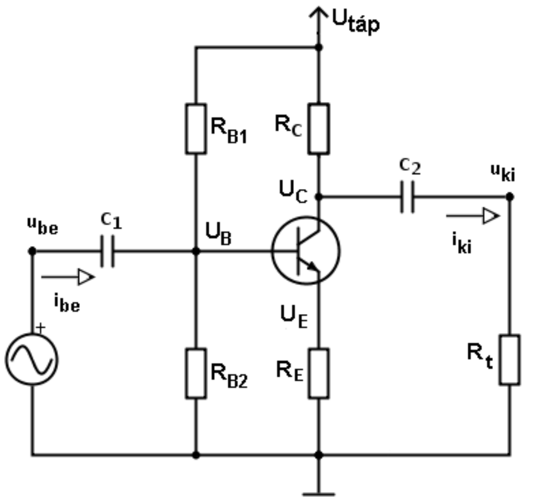
$$g_{21} = \frac{I_C}{V_T}$$

$$A_{V\max} = -g_{21} R_C = -\frac{I_C R_C}{V_T} = -\frac{V_{supply}}{2V_T}$$

We can see that with these conditions, values of I_C and R_C are not independent, and the max gain is dependent only on power supply.

Gain can be greater than this, by the way - if we don't use the max range condition for VC - for example if our input signal is very small, so its amplified value is still below the max output, then max range doesn't matter as much and we can have greater gain.
Increasing R_C increases output resistance, which can be problematic. Increasing I_C increases power dissipation.

1.4.2. CE amp with emitter capacitor removed



27.

Taking CE out of the circuit, the emitter resistor's negative feedback effect will work in AC as well. Thus it will try to keep IC from changing. But while very low-frequency changes are unwanted, we do want changes in the frequency of the input. So suppressing these means there will be very little useful AC signal on the output, ie. the gain is suppressed as well.

We can make an estimation on the voltage gain.

Now the input signal v_{in} fall only partly on BE junction, and partly on R_E. Remember the case of the diode+resistor. When the V_B is large enough compared to 0,7V, the majority of the change of voltage (ie. v_{in}) will fall on R_E, and V_{BE} is approximately constant, ie. v_{BE} is very small, close to zero in AC. Thus in AC we can say $v_E \approx v_{in}$.

$$v_{in} = v_{BE} + v_E \approx v_E$$

$$i_E = \frac{v_E}{R_E} = \frac{v_{be}}{R_E}$$

$$i_C \approx i_E$$

$$v_{out} = -i_C R_C$$

$$A_V = \frac{v_{out}}{v_{in}} \approx \frac{-i_C R_C}{v_{in}} = \frac{-\frac{v_{in}}{R_E} R_C}{v_{in}} = -\frac{R_C}{R_E}$$

if loaded:

$$A_v = -\frac{R_C \| R_L}{R_E}$$

This is much smaller than the gain when C_E is present. (As the expression "negative feedback" should imply.)

This formula is only true as long as the initial assumption is true, that v_{BE} is small. If R_E is very small, for example, this will not hold true. The resulting gain can not be greater than that of the original circuit. So choosing for example R_C=5kΩ and R_E=5Ω would not make a thousand times gain.

The input resistance also changes. At first sight, we may say that R_E parallel to R_L is added to r_{BE}, but closer inspection tells us otherwise.

$$r_{in} = R_{B1} \parallel R_{B2} \parallel r'_{in}$$

$$r'_{in} = \frac{v_{in}}{i_B}$$

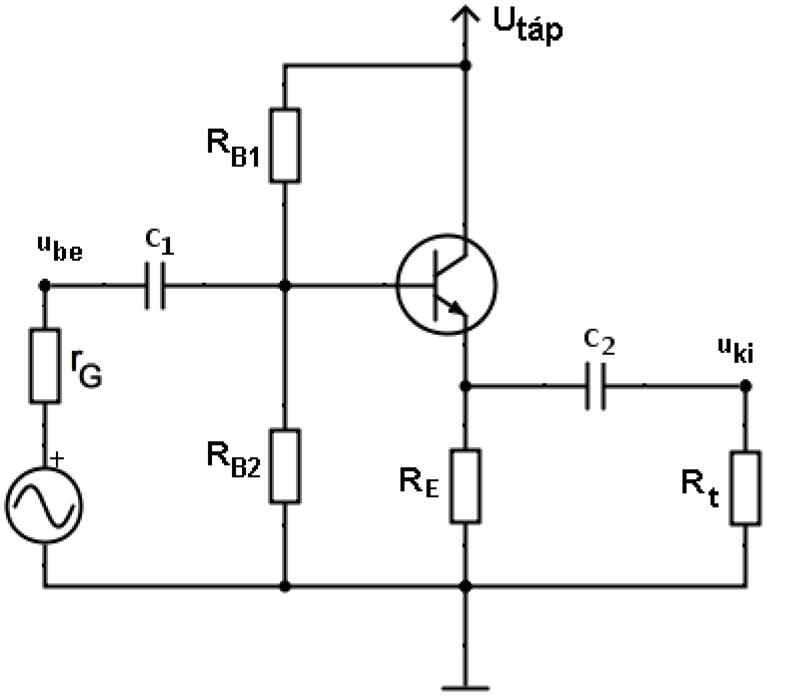
$$v_{in} = u_{BE} + u_E$$

$$r'_{in} = \frac{v_{BE} + v_E}{i_B} = \frac{v_{BE}}{i_B} + \frac{v_E}{i_B} = r_{BE} + \frac{v_E \beta}{i_C} = r_{BE} + \beta R_E$$

$$r_{in} = R_{B1} \parallel R_{B2} \parallel (r_{BE} + \beta R_E)$$

Because R_E has beta times the current compared to the input where we look at the resistance, the R is also seen as beta times greater. (Strictly speaking B+1 times, but the difference is negligible if B is large enough.)

1.4.3. Common Collector amplifier (CC)



28. CC amplifier

This amplifier has less than one voltage gain. It is used as an end stage amplifier to connect to low impedance loads. Connecting those to the high output resistance CE amp would lead to low power. The CC has high input resistance and low output resistance, thus makes it possible to connect to low impedance loads and have high power on the output.
(Think about, for example, loudspeakers of 4 or 8 or 16 etc ohms of impedance...)

Voltage gain:

$$g_{21} = \frac{i_C}{u_{BE}} \quad \Rightarrow \; i_C = g_{21} u_{BE}$$

$$v_{out} = v_E = i_E (R_E \| R_L) \approx i_C (R_E \| R_L) = g_{21} v_{BE} (R_E \| R_L)$$

$$v_{in} = v_{BE} + v_E = v_{BE} + g_{21} v_{BE} (R_E \| R_L) = v_{BE} \left(1 + g_{21}(R_E \| R_L)\right)$$

$$AV = \frac{v_{out}}{v_{in}} = \frac{g_{21} v_{BE} (R_E \| R_L)}{v_{BE}\left(1 + g_{21}(R_E \| R_L)\right)} = \frac{g_{21}(R_E \| R_L)}{1 + g_{21}(R_E \| R_L)}$$

If eg. I_C=1mA and R_E=5kΩ, the unloaded gain is A_V=0,995.
If a load of R_L=26Ω is used, A_V=0,5. (Similar values to 0,5 occur when load is similar to the output resistance, ie. we have power matching.)

The input impedance is similar to the CE without C_E.

$$r_{in} = R_1 \| R_2 \| \left(r_{BE} + \beta(R_E \| R_L)\right)$$

For output resistance, we invert our previous logic. If the higher current output resistors seem beta times higher from the input, then the input resistors seem beta times smaller from the output. The transistor's BE junction's dynamic resistance is also different, as now we look at it from emitter side, so instead of r_{BE} we use r_{EB}.

$$r_{EB} = \frac{v_{BE}}{i_E} \approx \frac{v_{BE}}{i_C} = \frac{r_{BE}}{\beta} = \frac{1}{g_{21}}$$

$$r_{out} = R_E \,\|\, \left(r_{EB} + \frac{1}{\beta}\left(R_{B1} \,\|\, R_{B2} \,\|\, r_G\right) \right) = R_E \,\|\, \left(\frac{1}{\beta}\left(r_{BE} + R_{B1} \,\|\, R_{B2} \,\|\, r_G\right) \right)$$

Here the output resistance of the previous circuit or function generator, r_G is also seen. For a lab function generator r_G is usually 50Ω. It could also be a CE amplifier, with a few kΩ output resistance. Divided by beta, it's still a small number. Therefore r_{out} is relatively small here.

Usually R_{B1} and $R_{B2} >> r_G$, and $R_E >> (r_{EB}+r_G/\beta)$, thus

$$r_{out} = R_E \,\|\, \left(r_{EB} + \frac{1}{\beta}\left(R_{B1} \,\|\, R_{B2} \,\|\, r_G\right) \right) \approx r_{EB} + \frac{r_G}{\beta}$$

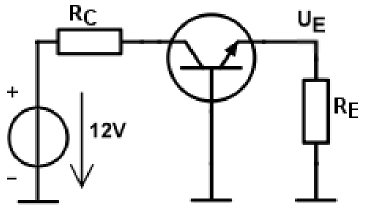
If we take our previous CE circuits output resistance as r_G=5kΩ and β=200:

$$r_{EB} = \frac{U_T}{I_C} = 26\Omega$$

$$r_{out} \approx r_{EB} + \frac{r_G}{\beta} = 26\Omega + 25\Omega = 51\Omega$$

1.5. Tricky questions

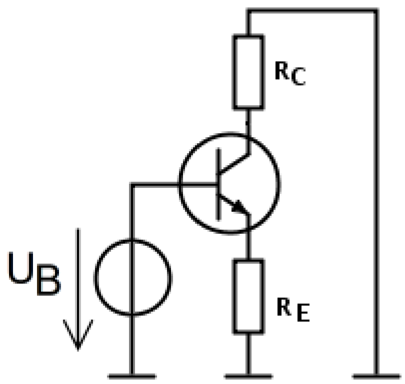
1.



29.

Find I_E , I_C , V_C and V_E . (R_C=R_E=1kΩ)

2.



30.

Find I_C and I_E áramait (draw the arrows as well). V_B=5V; R_E=R_C=1kΩ.

3.

What happens if the Zener diode is used instead of RB2 in the CE amplifier?

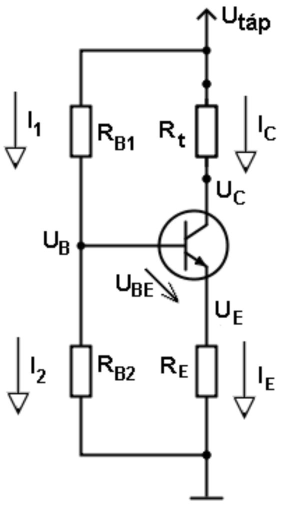
1.6. Examples

I.

Exercise: Design a BJT current generator for ohmic load!

Parameters: V_0=12V ; I_C=2mA ; B=200 ; V_{CEsat}=0,2V

Solution:



If RLmxa is not given, we can choose VE ourselves. A few volts is usually enough. Let's now choose V_E=2V. V_{BE} can be treated as 0,7V. For B we used a minimum value listed in datasheet (taking a modern general low current transistor).

$$V_E = 2V$$

$$V_B = V_E + V_{BE} = 2,7V$$

$$I_E \approx I_C = 2mA$$

$$R_E = \frac{V_E}{I_E} = \frac{2V}{2mA} = 1mA$$

In practice choosing $I_1=10\cdot I_B$ usually works.

$$I_1 = 10I_B = 100\mu A$$

$$I_2 = 9I_B = 90\mu A$$

$$R_{B1} = \frac{V_0 - V_B}{I_1} = \frac{12V - 2,7V}{100\mu A} = 93k\Omega$$

$$R_{B2} = \frac{V_B}{I_2} = \frac{2,7V}{90\mu A} = 30k\Omega$$

The load's maximum value can be calculated if we know V_{CEsat}. It is often not known precisely in advance. We can take its value to be a few hundred mV when I_C is a few mA. Let's use 0,2V now.

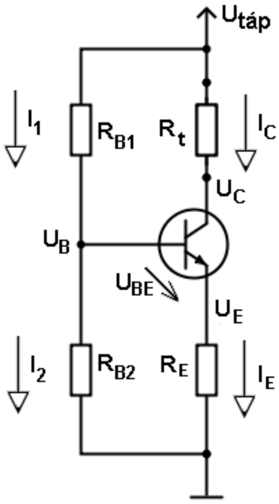
$$R_{L\_max} = \frac{V_0 - V_{CEsat} - V_E}{I_C} = \frac{12V - 0,2V - 2V}{2mA} = 4,9k\Omega$$

II.

Exercise: Design a BJT current generator for ohmic load!

Parameters: U_0=5V ; I_C=100μA ; B=200; V_{CEsat}=0,2V; R_{Lmax}=30kΩ

Solution:



R_{Lmax} is now given, thus V_E can not be freely chosen.

$$R_{L\max} = \frac{V_0 - V_{CEsat} - V_E}{I_C}$$

$$V_E = V_0 - V_{CEsat} - I_C \cdot R_{t\max} = 5V - 0,2V - 100\mu A \cdot 30k\Omega = 1,8V$$

(Note: R_{Lmax} can be treated as a minimum requirement, we can design for somewhat larger as well.)

$$V_B = V_E + V_{BE} = 2,5V$$

$$I_B = \frac{I_C}{B} = \frac{100\mu A}{200} = 0,5\mu A$$

$$I_E \approx 100\mu A$$

$$R_E = \frac{V_E}{I_E} = \frac{1,8V}{100\mu A} = 18k\Omega$$

$$I_1 = 10I_B = 5\mu A$$
$$I_2 = 9I_B = 4{,}5\mu A$$
$$R_{B1} = \frac{V_0 - V_B}{I_1} = \frac{5V - 2{,}5V}{5\mu A} = 500k\Omega$$
$$R_{B2} = \frac{V_B}{I_2} = \frac{2{,}5V}{4{,}5\mu A} = 555k\Omega$$

Variation for base divider:

When the base current is so small as here, we may choose greater I_1 than the $10I_B$.

For example choose $500I_B$.

$$I_1 = 500I_B = 250\mu A$$
$$I_2 = 499I_B = 249{,}5\mu A$$
$$R_{B1} = \frac{V_0 - V_B}{I_1} = \frac{5V - 2{,}5V}{250\mu A} = 10k\Omega$$
$$R_{B2} = \frac{V_B}{I_2} = \frac{2{,}5V}{249{,}5\mu A} = 10{,}02k\Omega \approx 10k\Omega = \frac{2{,}5V}{250\mu A}$$

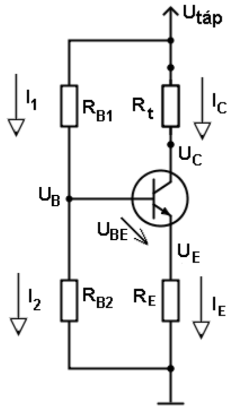
The result for R_{B2} is well within the normal tolerance of a 10kΩ resistor (because here we had 499 vs 500 difference instead of 9 vs 10 in the currents). Thus here we can safely say R_{B2}=10kΩ. As the base potential was set up to be half power supply, we can just take two equal resistors of a few kΩ and not even have to calculate much.

III.

Exercise: Calculate all voltages and currents and R_{Lmax}!

Parameters: V_0=15V ; B=200; R_{B1}= 100kΩ; R_{B2}=50kΩ; R_E=2,2kΩ; V_{CEsat}=0,2V

Solution:



Here we can not use the rule $I_1=10\cdot I_B$, as it is a design method, not a physical law. Also, even if we found out the value of I_B, we can't find I_C from saying $I_C=BI_B$, as value of B is very uncertain.

We estimate base potential:

Suppose: $I_1 >> I_B$, so $I_1 \approx I_2$

$$V_B = V_0 \frac{R_{B2}}{R_{B1}+R_{B2}} = 15V \cdot \frac{50k\Omega}{100k\Omega + 50k\Omega} = 5V$$

Let's use V_{BE}=0,7V.

$$V_E = V_B - 0,7V = 4,3V$$

$$I_E = \frac{V_E}{R_E} = \frac{4,3V}{2,2k\Omega} = 1,95mA$$

$$B > 100 \to I_C = I_E = 1,95mA$$

$$R_{L\_max} = \frac{V_0 - V_{CEsat} - V_E}{I_C} = \frac{15V - 0,2V - 4,3V}{1,95mA} = 5,37k\Omega$$ [3]

Check our initial estimation:

$$I_2 = \frac{V_B}{R_{B2}} = \frac{5V}{50k\Omega} = 0,1mA$$

$$I_B = \frac{I_C}{B} = \frac{1,95mA}{200} \approx 0,01mA$$

$$I_2 \geq 10 \cdot I_B$$

Thus our estimation is not bad.

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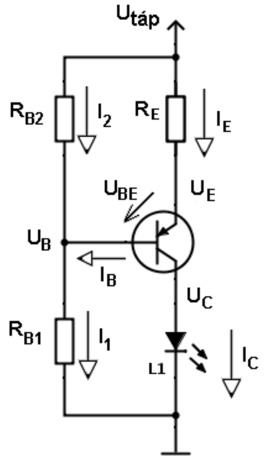
[3] Before you say that 5,38k is the correct value: the previous values were written down in rounded form, but saved in more precise form in the calculator's memory..

IV.

Exercise: Design a BJT current generator with PNP transistor for driving a white LED!
Parameters: I_{LED}=400mA ; V_{LED}=3V ; B=200 ; V_{CEsat}=0,2V ; $V_{EB}(I_C$=400mA)=0,9V

Solution:



For PNP transistors, all voltage and current arrows are reversed. Therefore we usually connect them with emitter towards the supply. The forward bias needs emitter to be more positive than the base.

I chose a transistor where the V_{EB} is listed as 0,9V for 400mA I_C. (The current is high, probably a bit high for this transistor, but should work.)

Here the power supply voltage is not specified, we have to choose it. Data sheet says V_{CEsat} is about 0,2V at 400mA I_C. Let us design with safety margin, give V_{EC}=1V to get $V_E=V_{LED}+V_{EC}$=4V.
If we use the commonly found 5V supply then we still have V_{RE}=1V, which is acceptable, thus the 5V supply is also acceptable.

$$V_0 = 5V$$
$$V_{EB} = 0,9V$$
$$V_C = V_{LED} = 3V$$
$$V_{RE} = 1V$$
$$V_E = V_0 - V_{RE} = 4V$$
$$I_B = \frac{I_C}{B} = \frac{400mA}{200} = 2mA$$
$$I_E = I_C + I_B = 402mA \approx 400mA$$
$$R_E = \frac{V_{RE}}{I_E} = \frac{2V}{400mA} = 5\Omega$$
$$V_B = V_E - V_{EB} = 3,1V$$
$$I_1 = I_2 + I_B$$
$$I_1 = 10I_B = 20mA$$
$$I_2 = 9I_B = 18mA$$
$$R_{B1} = \frac{V_B}{I_1} = \frac{3,1V}{20mA} = 155\Omega$$
$$R_{B2} = \frac{V_0 - V_B}{I_2} = \frac{1,9V}{18mA} = 105,5\Omega$$

When current is more than about 100mA, it is advisable to calculate with powers as well.

$$P_{LED} = V_{LED} \cdot I_C = 3V \cdot 400mA = 1,2W$$
$$P_{RE} = V_{RE} \cdot I_E = 1V \cdot 400mA = 400mW$$
$$P_{transistor} \approx V_{EC} \cdot I_C = 1V \cdot 400mA = 400mW$$

This means we should use a resistor designed for minimum 0,5W power as RE. The transistor datasheet mentions 700mW max power, so it is good, but probably requires a heat sink.
The LED parameters are from its datasheet's default values, so should be right as well, but cooling is required here.

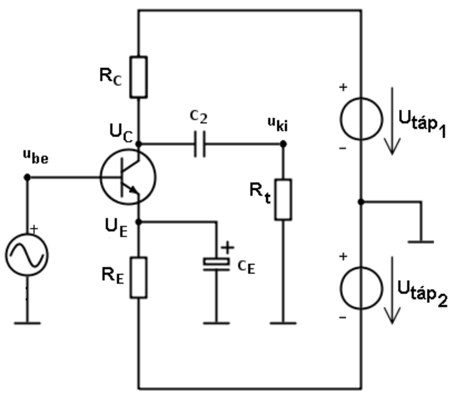
I chose (somewhat randomly) a 12A02CH transistor and a Cree XLamp XP-G LED for this exercise.

V.

Exercise: Calculate parameters of this CE amp!

Parameters: V_1=10V ; V_2=5V ; R_C=2,49kΩ; R_E=2,15kΩ ; R_L=5kΩ ; C_E=100µF ; C2=10µF ; B=400

Solution:



$$V_B = 0V$$
$$V_{BE} = 0,7V$$
$$V_E = V_B - V_{BE} = -0,7V$$
$$V_{RE} = V_E - (-V_{táp2}) = 4,3V$$
$$I_E = \frac{V_{RE}}{R_E} = \frac{4,3V}{2150\Omega} = 2mA$$
$$I_C \approx I_E = 2mA$$
$$I_B = \frac{I_C}{B} = 5\mu A$$
$$V_C = V_1 - I_C R_C = 10V - 2mA \cdot 2,49k\Omega = 5,02V \approx 5V$$
$$r_B = \frac{V_T}{I_B} = \frac{26mV}{5\mu A} = 5,2k\Omega$$
$$r_{in} = r_B = 5,2k\Omega$$
$$r_{out} \approx R_C = 2,49k\Omega$$
$$g_{21} = \frac{I_C}{V_T} = \frac{2mA}{26mV} = 77mS$$
$$A_V = -g_{21}(r_{out} \times R_t) = -g_m(R_C \times R_t) = -77mS \cdot \frac{2490 \cdot 5000}{2490 + 5000} \approx -128$$
$$a(dB) = 20lg\left|A_U\right| = 42dB$$
$$A_I = A_V \frac{r_{in}}{R_L} = -128 \cdot \frac{5200}{5000} = -133$$